Vieta's Formulas Handout

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1 Problems

1. The zeroes of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

2. The polynomial $x^3 - ax^2 + bx - 2010$ has three positive integer roots. What is the smallest possible value of a?

3. Let P be a cubic monic polynomial with roots a, b, and c. If $P(1) = 91$ and $P(-1) = -121$, compute the maximum possible value of

$$\frac{ab + bc + ca}{abc + a + b + c}.$$

4. Let a, b, and c be the 3 roots of $x^3 - x + 1 = 0$. Find $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}$.

5. Find the sum of all the real values of x satisfying $(x + \frac{1}{x} - 17)^2 = x + \frac{1}{x} + 17$.

6. The sum of the squares of the roots of the equation $x^2 + 2hx = 3$ is 10. Compute the absolute value of h.

7. Find the sum of the real roots of the polynomial

$$\prod_{k=1}^{100}(x^2 - 11x + k) = (x^2 - 11x + 1)(x^2 - 11x + 2)...(x^2 - 11x + 100).$$

8. The roots of the polynomial $P(x) = x^3 + 5x + 4$ are r, s, and t. Evaluate $(r+s)^4(s+t)^4(t+r)^4$.

9. Let x and y be real numbers with $x > y$ such that $x^2y^2 + x^2 + y^2 + 2xy = 40$ and $xy + x + y = 8$. Find the value of x.

10. The polynomial $x^4 + ax^3 + bx^2 + cx + d$ has a root at $x = 0$ and a double root at $x = -2$. What is the value of d?

11. Find the sum of the squares of the roots of the polynomial $x^2 + 2x - 1$.

12. Let r_1, r_2, r_3 be the roots of $f(x) = x^3 - 6x^2 - 5x + 22$. Find $\dfrac{1}{r_1r_2} + \dfrac{1}{r_2r_3} + \dfrac{1}{r_3r_1}$.

13. Let x be the answer to this question. What is the sum of possible values of the average of the numbers $\{-16x, 3x^2, 10x\}$?

14. Let r, s, and t be the three roots of the equation $8x^3 + 1001x + 2008 = 0$. Find

$$(r+s)^3 + (s+t)^3 + (t+r)^3.$$

15. The function $f(x) = x^5 - 20x^4 + ax^3 + bx^2 + cx + 24$ has the interesting property that its roots can be arranged to form an arithmetic sequence. Determine $f(8)$.

2 Sources

1. 2015 AMC 10A Problem 23
2. 2010 AMC 10A Problem 21
3. National Internet Math Olympiad Summer Contest 2017 Problem 9
4. 2009 February Harvard MIT Math Tournament Algebra Problem 5
5. 2015 Purple Comet High School Problem 10
6. 1971 American High School Math Exam Problem 20
7. National Internet Math Olympiad Summer Contest 2013 Problem 4
8. National Internet Math Olympiad Summer Contest 2011 Problem 9
9. 2013 February Harvard MIT Math Tournament Algebra Problem 1
10. 2014 Berkeley Math Tournament Fall Speed Round Problem 38
11. 2014 Berkeley Math Tournament Fall Speed Round Problem 42
12. 2013 Berkeley Math Tournament Fall Speed Round Problem 89
13. 2013 Berkeley Math Tournament Fall Speed Round Problem 90
14. 2015 Berkeley Math Tournament Spring Individual Round Problem 11
15. 2014 Berkeley Math Tournament Spring Analysis Round Problem 4