Number Bases Handout

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1 Problems

1. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

2. Convert 731_8 to a base-4 number.

3. What is the largest base 10 number that can be expressed in 3 digits of base 16?

4. What is the base 6 representation of the base 10 number 10?

5. What is the base 8 number 1225 in base 10?

6. Evaluate $11_2 + 111_3 + 1111_4$ and express as a base 5 number.

7. In what base b with $b > 7$, is 3 times 7 equal to 18?

8. When 2017 (in base 10) is expressed in base 7, what is the base-10 average of those base-7 digits?

9. N is the smallest positive integer for which $275 \cdot N$ (base 10) is a perfect cube. Write N in octal.

10. Let A, B, C be digits, $x_{10} = ABC_4$, and $y_{10} = ABC_6$. If $y = 2x$, and the sum of all possible values of x is z, find the sum of the digits of z.

11. The fraction $\dfrac{103_b}{136_b}$ can be reduced to $\dfrac{14_b}{18_b}$ (which is simplest since it is irreducible in base 10). What is the simplest form of the fraction $\dfrac{149_b}{338_b}$, written as a quotient of integers, each in base b?

12. When the decimal number 271 is converted to base 4, what is the sum of the digits of the base four number, expressed in base ten?

13. In base eleven, the letter A is the symbol for 10. What is $1A9_{11}$ expressed in base ten?

14. The number A, when expressed in base 7, is $0.222..._7$. When expressed in base 10, A can be written as $0.ppp..._{10}$. What is the value of p?

15. The four digit number $X45Y_{12}$ is divisible by 143_{10}. Evaluate $X + Y$ in base 10.

2 Sources

1. 2013 AMC 10A Problem 23
2. 2014-2015 Log1 Contest Theta Individual Problem 3
3. 2007-2008 Log1 Contest Theta Individual Problem 7
4. 2008-2009 Log1 Contest Theta Number Theory Problem 5

5. 2008-2009 Log1 Contest Theta Number Theory Problem 9
6. 2011-2012 Log1 Contest Theta Number Theory Problem 4
7. 2016-2017 National Mu Alpha Theta Open Number Theory Problem 1
8. 2016-2017 National Mu Alpha Theta Open Number Theory Problem 6
9. 2016-2017 National Mu Alpha Theta Open Number Theory Problem 15
10. 2015-2016 National Mu Alpha Theta Open Number Theory Problem 9
11. 2015-2016 National Mu Alpha Theta Open Number Theory Problem 14
12. 2014-2015 National Mu Alpha Theta Open Number Theory Problem 5
13. 2014-2015 National Mu Alpha Theta Open Number Theory Problem 8
14. 2014-2015 National Mu Alpha Theta Open Number Theory Problem 18
15. 2012-2013 National Mu Alpha Theta Open Number Theory Problem 22