Geometry Handout 2

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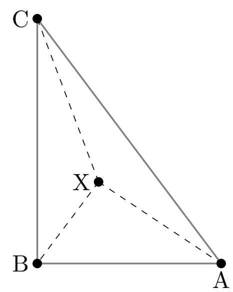
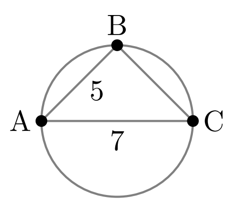
1 Problems

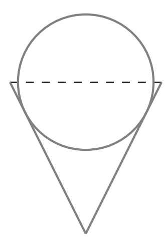
1. Let $\triangle ABC$ be a triangle such that $\overline{AB} = 3$, $\overline{BC} = 4$, and $\overline{AC} = 5$. Let X be a point in the triangle. Compute the minimal possible value of $\overline{AX}^2 + \overline{BX}^2 + \overline{CX}^2$.



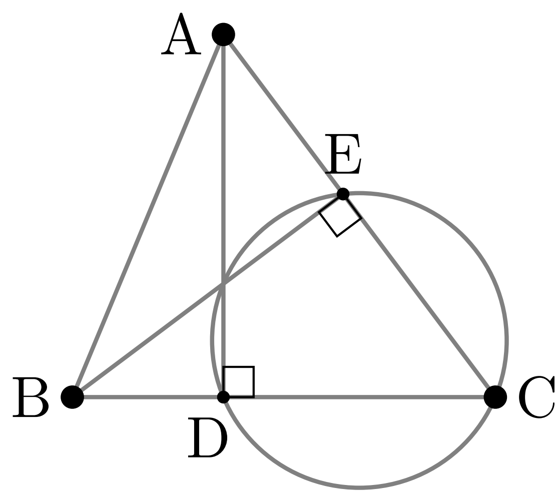
2. In a circle, chord $\overline{AB}$ has length 5 and chord $\overline{AC}$ has length 7. Arc $\overset{\frown}{AC}$ is twice the length of arc $\overset{\frown}{AB}$ and both arcs have angles less than 180 degrees. Compute the area of the circle.



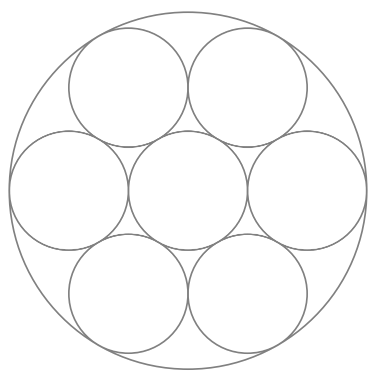
3. Spencer eats ice cream in a right circular cone with an opening of radius 5 and a height of 10. If Spencers ice cream scoops are always perfectly spherical, compute the radius of the largest scoop he can get such that at least half of the scoop is contained within the cone.



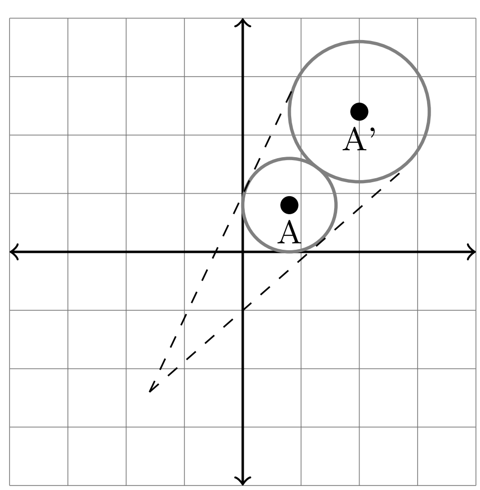
4. Let $\triangle ABC$ be a triangle with $\overline{AB} = 13$, $\overline{BC} = 14$, and $\overline{AC} = 15$. Let D and E be the feet of the altitudes from A and B, respectively. Find the circumference of the circumcircle of triangle $\triangle CDE$.



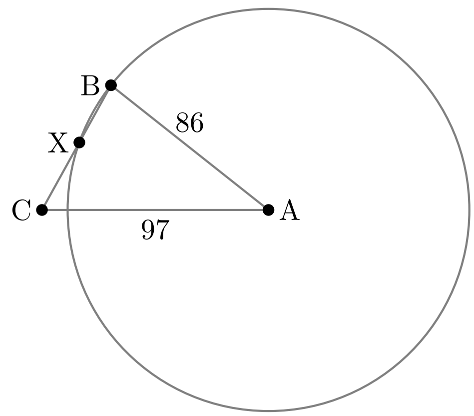
5. Seven cookies of radius 1 inch are cut from a circle of cookie dough, as shown. Neighboring cookies are tangent, and all except the center cookie are tangent to the edge of the dough. The leftover scrap is reshaped to form another cookie of the same thickness. What is the radius in inches of the scrap cookie?



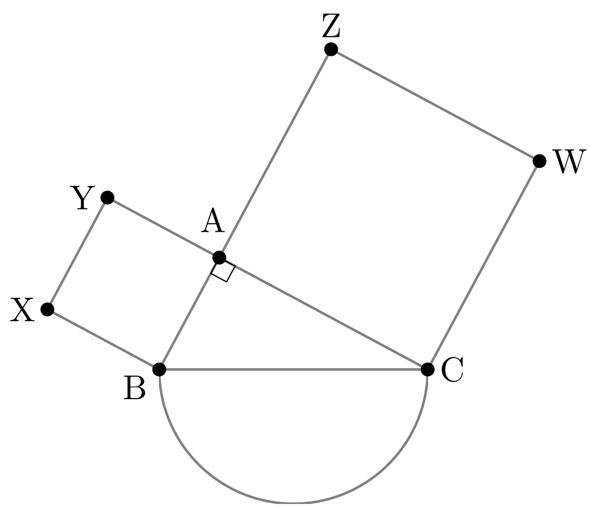
6. A dilation of the plane - that is, a size transformation with a positive scale factor - sends the circle of radius 2 centered at A $(2, 2)$ to the circle of radius 3 centered at A' $(5, 6)$. What distance does the origin O $(0, 0)$, move under this transformation?



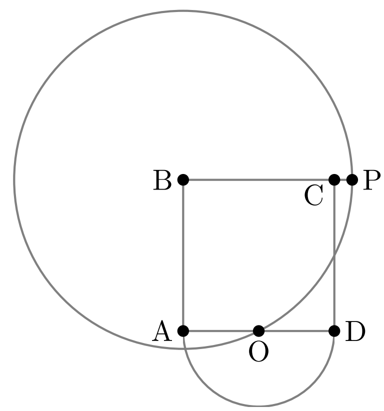
7. In triangle $\triangle ABC$, AB = 86, and $\overline{AC} = 97$. A circle with center A and radius $\overline{AB}$ intersects $\overline{BC}$ at points B and X. Moreover, $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is $\overline{BC}$?



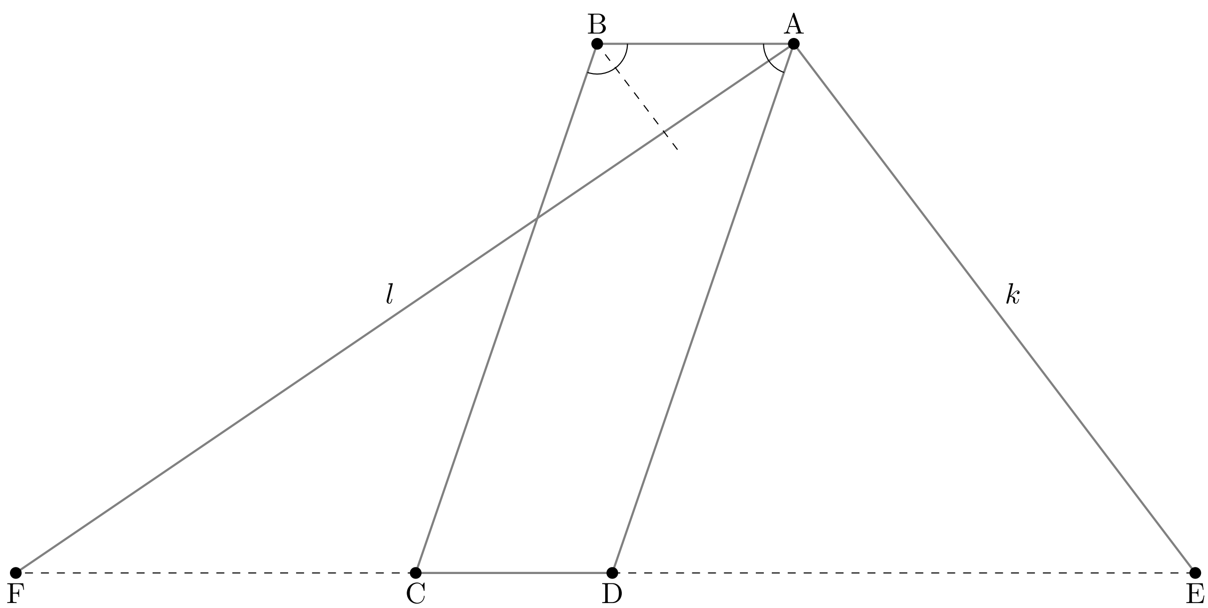
8. Right triangle $\triangle ABC$ has squares $ABXY$ and $ACWZ$ drawn externally to its legs and a semicircle drawn externally to its hypotenuse $\overline{BC}$. If the area of the semicircle is 18π and the area of triangle $\triangle ABC$ is 30, what is the sum of the areas of squares $ABXY$ and $ACWZ$?



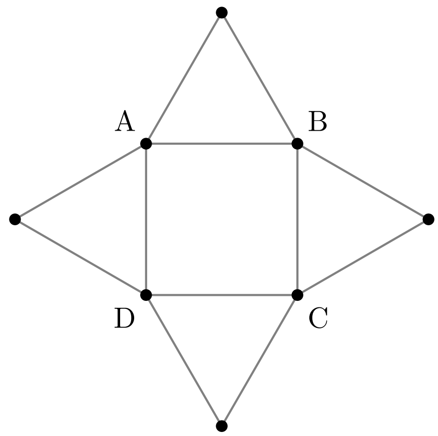
9. Let O be the center of a semicircle with diameter $\overline{AD}$ and area 2π. Given square $ABCD$ drawn externally to the semicircle, construct a new circle with center B and radius $\overline{BO}$. If we extend $\vec{BC}$, this new circle intersects $\vec{BC}$ at P. What is the length of $\overline{CP}$?



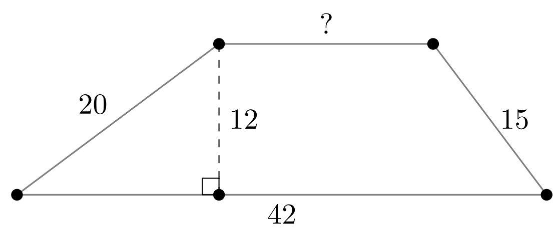
10. Consider a parallelogram $ABCD$. Let k be the line passing through A and parallel to the bisector of angle $\angle ABC$, and let l be the bisector of angle $\angle BAD$. Let k intersect line $\overline{CD}$ at E and l intersect line $\overline{CD}$ at F. If $\overline{AB} = 13$ and $\overline{BC} = 37$, find the length $\overline{EF}$.



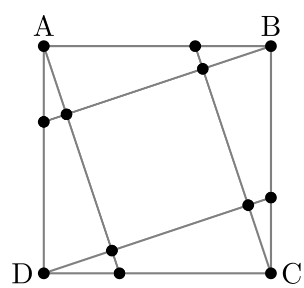
11. Square $ABCD$ has equilateral triangles drawn external to each side, as pictured. If each triangle is folded upwards to meet at a point E, then a square pyramid can be made. If the center of square $ABCD$ is O, what is the measure of angle $\angle OEA$?



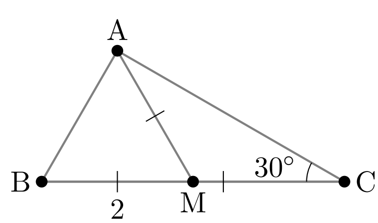
12. A trapezoid with height 12 has legs of length 20 and 15 and a larger base of length 42. What are the possible lengths of the other base?



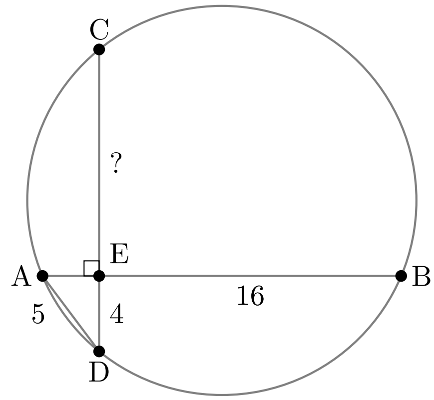
13. As pictured, lines are drawn from the vertices of a unit square to an opposite trisection point. If each triangle has legs with ratio 3 : 1, what is the area of the slanted square formed in the middle of $ABCD$?



14. Let $\triangle ABC$ be a triangle. Let M be the midpoint of $\overline{BC}$. Suppose $\overline{MA} = \overline{MB} = \overline{MC} = 2$ and angle $\angle ACB = 30°$. Find the area of the triangle.



15. Chords $\overline{AB}$ and $\overline{CD}$ of a given circle are perpendicular to each other and intersect at a right angle E. Given that $\overline{BE} = 16$, $\overline{DE} = 4$, and $\overline{AD} = 5$, find $\overline{CE}$.



2 Sources

1. 2014 Stanford Math Tournament General 25
2. 2014 Stanford Math Tournament Geometry 2
3. 2014 Stanford Math Tournament Geometry 3
4. 2014 Stanford Math Tournament Geometry 7
5. 2016 AMC 10A Problem 15
6. 2016 AMC 10B Problem 20
7. 2013 AMC 10A Problem 23
8. Berkeley Math Tournament Fall 2012 Individual Problem 4
9. Berkeley Math Tournament Fall 2012 Individual Problem 9
10. Berkeley Math Tournament Fall 2012 Individual Problem 19
11. Berkeley Math Tournament Fall 2012 Team Problem 3
12. Berkeley Math Tournament Fall 2012 Team Problem 5
13. Berkeley Math Tournament Fall 2012 Problem 8
14. Berkeley Math Tournament Fall 2013 Problem 17
15. Berkeley Math Tournament Fall 2013 Speed Problem 75