Geometry Handout 3 Answers and Solutions

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November 14, 2017

1 Answers

1. $\frac{\pi\sqrt{3}}{100}$

2. $\frac{31\sqrt{3}}{5}$

3. $3 - 2\sqrt{2}$

4. 61°

5. 27

6. $\frac{39}{67}$

7. $\frac{\pi}{4}$

8. $\sqrt{2}$

9. $\frac{\sqrt{6}}{2}$

10. 51

11. $\sqrt{3}$

12. $\frac{504}{1007}$

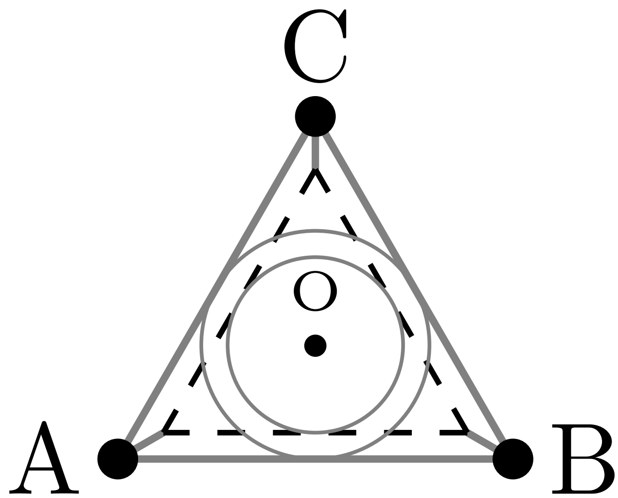
13. $\frac{3\sqrt{14}}{5}$

14. 130π

15. 252

2 Problems

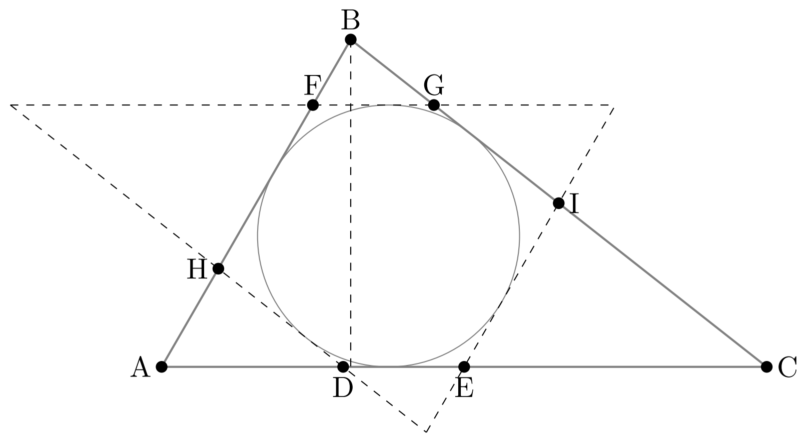
1. Let triangle $\triangle ABC$ be an equilateral triangle with height 13, and let O be its center. Point X is chosen at random from all points inside triangle $\triangle ABC$. Given that the circle of radius 1 centered at X lies entirely inside triangle $\triangle ABC$, what is the probability that the circle contains O?



Solution: Notice that in order for the circle to contain O, X needs to be chosen within a distance of 1 from O. The area of the set of all points which are within 1 unit of O is the area of a circle of radius 1, which is $1^2\pi = \pi$. All of these points will be included in the set of possible points for X, as the height of $\triangle ABC$ is far greater than 1. Now we must determine the area of the set of points X such that the circle of radius centered around each of these points is contained completely within $\triangle ABC$. By symmetry, the set of points X which work must also be an equilateral triangle. In addition, the equilateral triangle containing the points X which satisfy this property must be at least 1 unit from $\triangle ABC$ in all directions. We can notice that if this new equilateral triangle has an inradius which is 1 less than that of $\triangle ABC$, then all points within the new equilateral triangle will be 1 unit away from $\triangle ABC$ in all directions. Given that the height of $\triangle ABC$ is 13, we know that the inradius of $\triangle ABC$ is $\frac{13}{3}$, as the inradius of an equilateral triangle is always $\frac{1}{3}$ of its height. Therefore, the inradius of our new equilateral triangle is $\frac{13}{3} - 1 = \frac{10}{3}$. It follows that the height of our new equilateral triangle is 10 and its base is $\frac{20\sqrt{3}}{3}$ by the properties of $30-60-90$ triangles. It follows that the area of the set of points X with this property is $10 \cdot \frac{20\sqrt{3}}{3} \cdot \frac{1}{2} = \frac{100\sqrt{3}}{3}$. It follows that our answer is

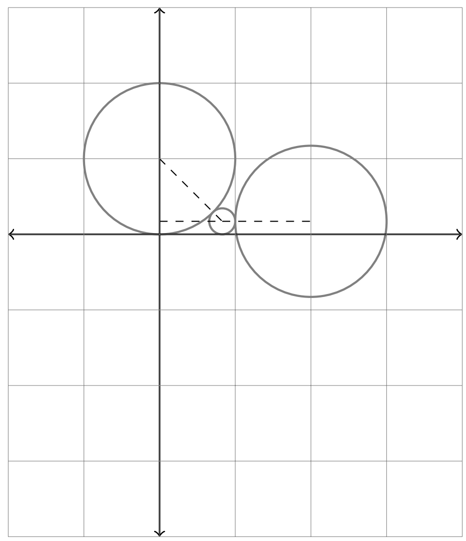
$$\frac{\pi}{\frac{100\sqrt{3}}{3}} = \boxed{\frac{\pi\sqrt{3}}{100}}$$

2. Triangle $\triangle ABC$ has $\overline{AB} = 5$, $\overline{BC} = 7$, and $\overline{CA} = 8$. New lines not containing but parallel to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are drawn tangent to the incircle of $\triangle ABC$. What is the area of the hexagon formed by the sides of the original triangle and the newly drawn lines?



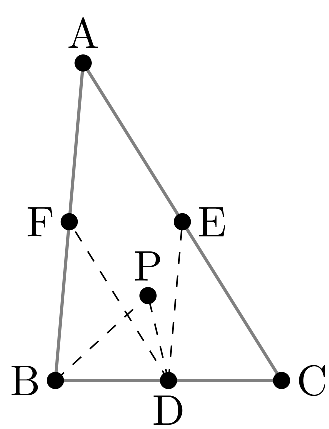
Solution: By Heron's Formula, the area of $\triangle ABC$ is $\sqrt{10 \cdot 2 \cdot 3 \cdot 5} = 10\sqrt{3}$. Using the fact that $A = \frac{a+b+c}{2}r$ where r is the inradius, we can find that the radius of the incircle is $\frac{10\sqrt{3}}{10} = \sqrt{3}$. Using the fact that $A = \frac{bh}{2}$, we can determine that the height from B to $\overline{AC}$ is $\frac{20\sqrt{3}}{8} = \frac{5\sqrt{3}}{2}$, the height from A to $\overline{BC}$ is $\frac{20\sqrt{3}}{7}$, and the height from C to $\overline{AB}$ is $\frac{20\sqrt{3}}{5} = 4\sqrt{3}$. Label the points as shown above. Consider $\triangle BFG$. We know that the height from B to $\overline{AC}$ is $\frac{5\sqrt{3}}{2}$. and we know the height from F to $\overline{AC}$ is $2r = 2\sqrt{3}$. Therefore, because $\overline{FG}$ is parallel to $\overline{AC}$, the height from B to $\overline{FG}$ is $\frac{5\sqrt{3}}{2} - 2\sqrt{3} = \frac{\sqrt{3}}{2}$. Because $\angle FBG \cong \angle ABC$ and $\angle BFG \cong \angle BAC$ as $\overline{FG} \parallel \overline{AC}$, we know by AA similarity that $\triangle BFG \sim \triangle BAC$. It follows because the height from B to $\overline{FG}$ is $\frac{\frac{\sqrt{3}}{2}}{\frac{5\sqrt{3}}{2}} = \frac{1}{5}$ of the height from B to $\overline{AC}$, the area of $\triangle BFG$ is $(\frac{1}{5})^2 = \frac{1}{25}$ of the area of $\triangle ABC$. Using similar methods with the other altitudes, we can find that the area of $\triangle AHD$ is $\frac{9}{100}$ of the area of $\triangle ABC$ and that the area of $\triangle IEC$ is $\frac{1}{4}$ of the area of $\triangle ABC$. It follows that the sum of the areas of these three triangles is $\frac{1}{25} + \frac{9}{100} + \frac{1}{4} = \frac{19}{50}$ of the area of $\triangle ABC$. Because the area we want is the area inside $\triangle ABC$ which does not contain these three triangles, the area of the hexagon is $1 - \frac{19}{50} = \frac{31}{50}$ of the area of $\triangle ABC$. It follows that the area of the hexagon is $\frac{31}{50}10\sqrt{3} = \boxed{\frac{31\sqrt{3}}{5}}$ as desired.

3. Two circles with radius one are drawn in the coordinate plane, one with center $(0, 1)$ and the other with center $(2, y)$ for some real number y between 0 and 1. A third circle is drawn so as to be tangent to both of the other two circles as well as the x-axis. What is the smallest possible radius for this third circle?



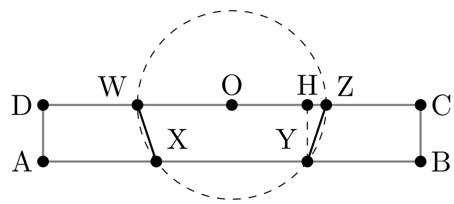
Solution: We can notice that the three circles must be in a configuration similar to the one above for the radius of the third circle to be minimized. Let the coordinates of the third circle be (x, r). We know by the Pythagorean Theorem on the segment connecting the two leftmost circles that $(r+1)^2 = x^2 + (1-r)^2$ or $x = 2\sqrt{r}$. In addition, we know that the rightmost point on the third circle must have an x-coordinate which is greater than or equal to 1, as otherwise it would be impossible for the rightmost circle to be tangent to the third circle. The rightmost point on the third circle has an x-coordinate of $x + r = 2\sqrt{r} + r$. It follows that $2\sqrt{r} + r \geq 1$, and rearranging we get $(\sqrt{r}+1)^2 \geq 2$, or $\sqrt{r} \geq \sqrt{2} - 1$. Squaring this inequality, we get $r \geq 3 - 2\sqrt{2}$. Equality will hold when $y = r$, as then the leftmost point on the rightmost circle will be $(1, r)$, the same as the rightmost point on the third circle. It follows that the minimum radius of the third circle is $\boxed{3 - 2\sqrt{2}}$.

4. Let $\triangle ABC$ be a triangle, and let D, E, and F be the midpoints of sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Let the angle bisectors of $\angle FDE$ and $\angle FBD$ meet at P.Given that $\angle BAC = 37°$ and $\angle CBA = 85°$, determine the degree measure of $\angle BPD$.



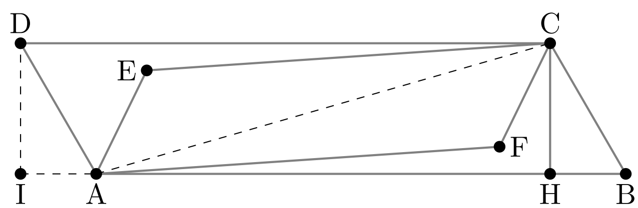
Solution: Notice that because $\angle ABC \cong \angle FBD$ and $\frac{\overline{FB}}{\overline{AB}} = \frac{\overline{DB}}{\overline{CB}} = \frac{1}{2}$, by SAS similarity, $\triangle FBD \sim \triangle ABC$. Similarly, $\triangle EDC \sim \triangle ABC$. It follows that $\angle BFD \cong \angle DEC \cong \angle BAC = 37°$. It follows that $\angle AFD \cong \angle AED = 180 - 37 = 143°$. Because the angles in any quadrilateral add up to 360°, it follows that $\angle FDE = 360 - 143 - 143 - 37 = 37°$. We know that $\angle ACB = 180 - \angle CAB - \angle ABC = 180 - 37 - 85 = 58°$. It follows that $\angle FDB = 58°$, and $\angle PDB = \frac{37}{2} + 58 = \frac{153}{2}°$. Finally, we know that $\angle PBD = \frac{85}{2}°$, and it follows that $\angle BPD = 180 - \frac{153}{2} - \frac{85}{2} = \boxed{61°}$ as desired.

5. $ABCD$ is a rectangle with $\overline{AB} = 20$ and $\overline{BC} = 3$. A circle with radius 5, centered at the midpoint of $\overline{DC}$, meets the rectangle at four points: W, X, Y, and Z. Find the area of quadrilateral $WXYZ$.



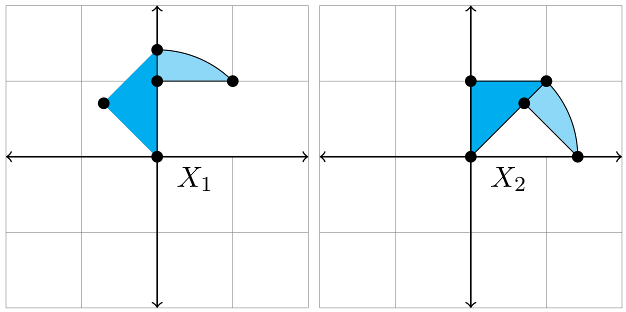
Solution: Let the foot of the perpendicular from Y to $\overline{CD}$ be H. By the Pythagorean Theorem, we have $\overline{HO} = \sqrt{\overline{OY}^2 - \overline{HY}^2} = \sqrt{25 - 9} = 4$. It follows that $\overline{XY} = 2 \cdot \overline{HO} = 8$ by symmetry. Because $WXYZ$ is a trapezoid, its area is $\frac{\overline{WZ} + \overline{XY}}{2}\overline{HY} = \frac{8+10}{2} \cdot 3 = \boxed{27}$ as desired.

6. $ABCD$ is a parallelogram satisfying $\overline{AB} = 7$, $\overline{BC} = 2$, and $\angle DAB = 120°$. Parallelogram $ECFA$ is contained in $ABCD$ and is similar to it. Find the ratio of the area of $ECFA$ to the area of $ABCD$.



Solution: Let the foot of the perpendicular from C to $\overline{AB}$ be H as shown. Because $\angle DAB = 120°$, we must have $\angle ABC = 180 - 120 = 60°$ due to the properties of parallelograms. It follows that $\triangle CHB$ is a $30 - 60 - 90$ triangle. It follows that $\overline{CH} = \sqrt{3}$, and that $\overline{HB} = 1$. It follows that $\overline{AH} = 7 - 1 = 6$, and $\overline{AC} = \sqrt{6^2 + 3} = \sqrt{39}$. Let the foot of the perpendicular from D to the extension of $\overline{AB}$ be I as shown. Notice that $\overline{DI} = \overline{CH} = \sqrt{3}$ and $\overline{IB} = \overline{AH} + 2\overline{HB} = 8$. It follows by the Pythagorean Theorem that $\overline{DB} = \sqrt{64 + 3} = \sqrt{67}$. Because segment $\overline{DB}$ in $ABCD$ must correspond to segment $\overline{AC}$ in $ECFA$, the ratio of their areas is $(\frac{\sqrt{39}}{\sqrt{67}})^2 = \boxed{\frac{39}{67}}$ as desired.

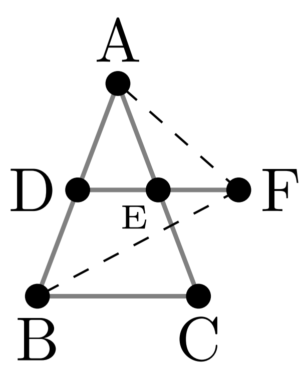
7. Plot points A, B, C at coordinates $(0, 0)$, $(0, 1)$, and $(1, 1)$ in the plane, respectively. Let S denote the union of the two line segments $\overline{AB}$ and $\overline{BC}$. Let X_1 be the area swept out when Bobby rotates S *counterclockwise* 45° about point A. Let X_2 be the area swept out when Calvin rotates S *clockwise* 45° about point A. Find $\frac{X_1 + X_2}{2}$.



Solution: Notice that the area of X_1 is the sum of the areas of an isosceles right triangle and a circular sector with another isosceles right triangle cut out. Notice that the hypotenuse of the right triangle that is included is the segment corresponding to the rotation of $\overline{AC}$. It follows that the right triangle that is included is equivalent to the right triangle that is excluded, and therefore the area of X_1 is a $45°$ sector of a circle with a radius of $\sqrt{1+1} = \sqrt{2}$. The area of this sector is $\pi \cdot (\sqrt{2})^2 \cdot \frac{45}{360} = \frac{\pi}{4}$. Notice that the area of X_2 is the result when the area of X_1 is rotated clockwise about the origin by $45°$. It follows that:

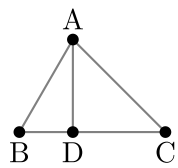
$$X_1 = X_2 \rightarrow \frac{X_1 + X_2}{2} = \frac{2X_1}{2} = X_1 = \boxed{\frac{\pi}{4}}$$

8. Let $\triangle ABC$ be an isosceles triangle with $\overline{AB} = \overline{AC}$. Let D and E be the midpoints of segments $\overline{AB}$ and $\overline{AC}$, respectively. Suppose that there exists a point F on ray $\overrightarrow{DE}$ outside of $\triangle ABC$ such that triangle $\triangle BFA$ is similar to triangle $\triangle ABC$. Compute $\frac{AB}{BC}$.



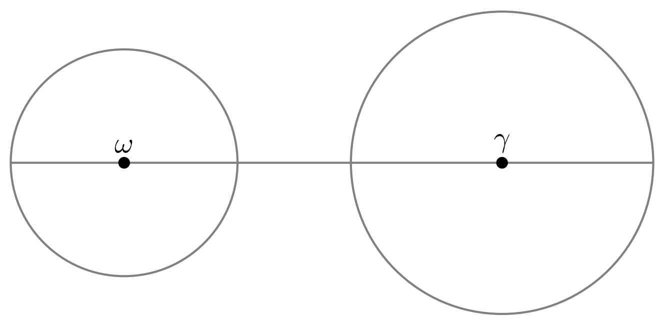
Solution: If $\triangle BFA \sim \triangle ABC$, then we must have that the ratio of their side lengths is $\frac{\overline{BA}}{\overline{AC}} = \frac{\overline{AC}}{\overline{AC}} = 1$. It follows that $\triangle BFA$ is congruent to $\triangle ABC$ as shown in the figure above. Let $\overline{AB} = 2x$ and let $\overline{BC} = 2y$. Notice that because $\triangle BFA$ is congruent to $\triangle ABC$, $\overline{DC} = \overline{DF}$. Notice that because $\angle BAC \cong \angle ABF$, we must have $\angle BAF \cong \angle BFA \cong \angle ACB \cong \angle ABC \cong \angle ADE$. It follows that $\triangle FAD$ is similar to triangle $\triangle ABC$. However, we know that $\overline{AF} = 2y$ and $\overline{AD} = x$. It follows that $\frac{2y}{x} = \frac{2x}{2y}$. Rearranging this equation, we get $\frac{x^2}{y^2} = 2$, or $\frac{x}{y} = \sqrt{2}$. However, this is equivalent to the ratio $\frac{\overline{AB}}{\overline{AC}} = \frac{2x}{2y}$, so our answer is $\boxed{\sqrt{2}}$.

9. Let $\triangle ABC$ be a triangle and D a point on $\overline{BC}$ such that $\overline{AB} = \sqrt{2}$, $\overline{AC} = \sqrt{3}$, $\angle BAD = 30°$, and $\angle CAD = 45°$. Find $\overline{AD}$.



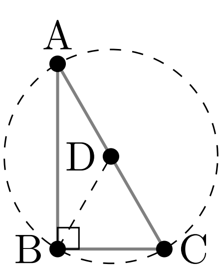
Solution: Let $\overline{AD} = x$, $\overline{BD} = l$, and $\overline{DC} = r$. Then by the Law of Cosines, $l^2 = 2 + x^2 - 2x\sqrt{2}\cos 30^\circ = 2 + x^2 - x\sqrt{6}$. Similarly, we know that $r^2 = 3 + x^2 - 2x\sqrt{3}\cos 45^\circ = 3 + x^2 - x\sqrt{6}$. It follows that $r^2 - l^2 = 1$. Notice that $r + l = \overline{DC} + \overline{DB} = \overline{BC}$. It follows by the Law of Cosines that $(l + r)^2 = 2 + 3 - 2\sqrt{6}\cos 75^\circ = 5 - (3 - \sqrt{3}) = 2 + \sqrt{3}$. It follows that $l + r = \sqrt{2 + \sqrt{3}} = \frac{\sqrt{3}+1}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$. Because $r^2 - l^2 = (r + l)(r - l) = 1$, we must have $r - l = \frac{1}{l+r} = \frac{\sqrt{6}-\sqrt{2}}{2}$. Now we know that $l + r = \frac{\sqrt{6}+\sqrt{2}}{2}$ and $r - l = \frac{\sqrt{6}-\sqrt{2}}{2}$. Solving this system of equations, we get $r = \frac{\sqrt{6}}{2}$ and $l = \frac{\sqrt{2}}{2}$. We know that $l^2 = 2 + x^2 - x\sqrt{6}$, so it follows that $2x^2 - 2x\sqrt{6} + 3 = 0$. Solving, we get $x = \boxed{\dfrac{\sqrt{6}}{2}}$ as desired.

10. Two circles ω and γ have radii 3 and 4 respectively, and their centers are 10 units apart. Let x be the shortest possible distance between a point on ω and a point on γ, and let y be the longest possible distance between a point on ω and a point on γ. Find the product xy.



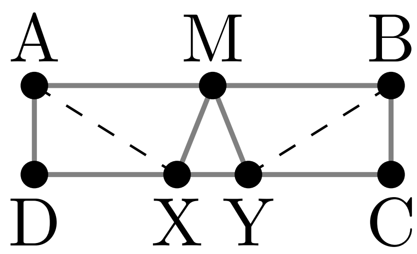
Solution: Notice that the shortest distance between points on the two different circles is the segment that lies between the circles on the segment connecting their centers. Similarly, the longest distance between points on the two different circles is the segment that lies on the segment connecting their centers and containing a diameter of each of the circles. It follows that $y = 10 - 3 - 4 = 3$ and $x = 10 + 3 + 4 = 17$. Therefore, $xy = 3 \cdot 17 = \boxed{51}$.

11. Let $\triangle ABC$ be a triangle with $\angle B = 90^\circ$. Given that there exists a point D on $\overline{AC}$ such that $\overline{AD} = \overline{DC}$ and $\overline{BD} = \overline{BC}$, compute the value of the ratio $\frac{AB}{BC}$.



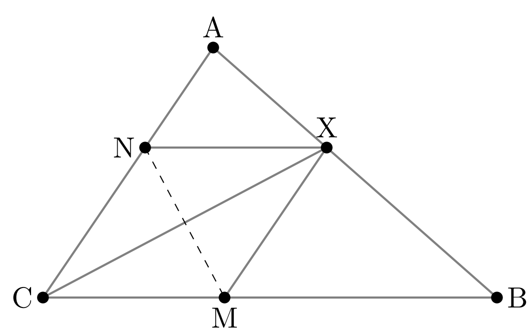
Solution: Notice that because D is the midpoint of $\overline{AC}$ and $\triangle ABC$ is a right triangle, D is the circumcenter of $\triangle ABC$. It follows that $\overline{DC} \cong \overline{DA} \cong \overline{DB} \cong \overline{BC}$. It follows that $\triangle BDC$ is an equilateral triangle and that $\angle ACB = 60°$. Therefore, $\frac{\overline{AB}}{\overline{BC}} = \tan 60° = \boxed{\sqrt{3}}$.

12. In rectangle $ABCD$ with area 1, point M is selected on $\overline{AB}$ and points X, Y are selected on $\overline{CD}$ such that $\overline{AX} < \overline{AY}$. Suppose that $\overline{AM} = \overline{BM}$. Given that the area of triangle $\triangle MXY$ is $\frac{1}{2014}$, compute the area of trapezoid $AXYB$. (Note: This diagram is not drawn to scale.)



Solution: Notice that the area of $\triangle MXY$ is $\frac{\overline{XY} \cdot \overline{AD}}{2}$. The area of $ABCD$ is $\overline{CD} \cdot \overline{AD}$. It follows that $\frac{\overline{XY}}{\overline{DC}} = \frac{\overline{XY} \cdot \overline{AD}}{\overline{DC} \cdot \overline{AD}} = \frac{2 \cdot \frac{1}{2014}}{1} = \frac{1}{1007}$. Therefore, $\overline{AB} + \overline{XY} = \overline{AB}(1 + \frac{1}{1007}) = \frac{1008}{1007}\overline{AB}$. It follows that the area of $AXYB$ is $\frac{(\overline{AB}+\overline{XY})}{2}\overline{AD} = \frac{1008}{1007} \cdot \frac{1}{2} \cdot (\overline{AB} \cdot \overline{AD}) = \boxed{\frac{504}{1007}}$.

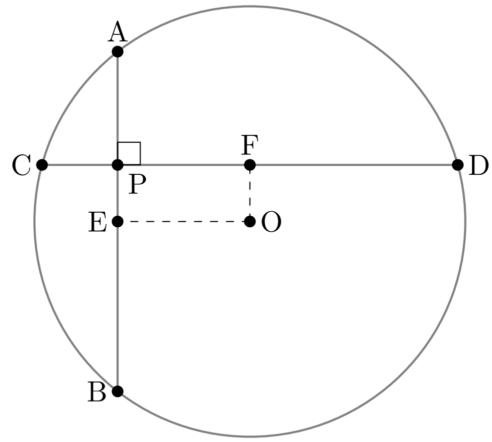
13. Let $\triangle ABC$ be a triangle with $\overline{AB} = 5$, $\overline{AC} = 4$, $\overline{BC} = 6$. The angle bisector of C intersects side $\overline{AB}$ at X. Points M and N are drawn on sides $\overline{BC}$ and $\overline{AC}$, respectively, such that $\overline{XM} \| \overline{AC}$ and $\overline{XN} \| \overline{BC}$. Compute the length $\overline{MN}$.



Solution: By the Angle Bisector Theorem, we have that $\frac{\overline{AX}}{\overline{XB}} = \frac{4}{6} = \frac{2}{3}$. Notice that because $\angle NAX \cong \angle CAB$ and $\angle ANX \cong \angle ACB$ because $\overline{NX} \parallel \overline{CB}$, $\triangle ANX \sim \triangle ACB$. It follows that $\frac{\overline{AN}}{\overline{AC}} = \frac{\overline{AX}}{\overline{AB}} = \frac{\overline{AX}}{\overline{AX}+\frac{3}{2}\overline{AX}} = \frac{2}{5}$. It follows that $\frac{\overline{CN}}{\overline{AC}} = 1 - \frac{2}{5} = \frac{3}{5}$. Similarly, we know $\triangle BXM \sim \triangle BAC$, and it follows that $\frac{\overline{BM}}{\overline{BC}} = \frac{\overline{BX}}{\overline{AB}} = \frac{3}{5}$. It follows that $\frac{\overline{CM}}{\overline{CB}} = 1 - \frac{3}{5} = \frac{2}{5}$. Notice by the Law of Cosines that $5^2 = 4^2 + 6^2 - 2 \cdot 4 \cdot 6 \cos \angle ACB$. Solving, we get $\cos \angle ACB = \frac{9}{16}$. We know that $\overline{CN} = \frac{3}{5} \cdot 4 = \frac{12}{5}$ and $\overline{CM} = \frac{2}{5} \cdot 6 = \frac{12}{5}$. It follows by the Law of Cosines that

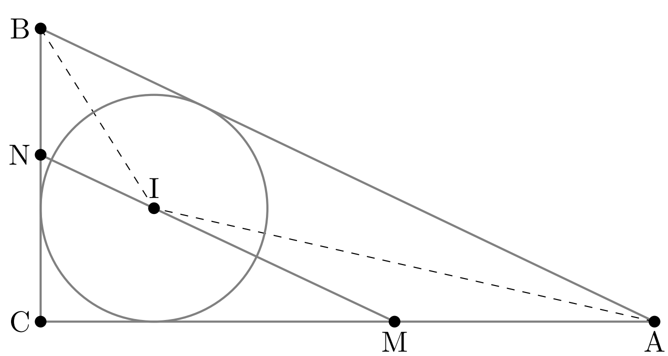
$$\overline{MN}^2 = (\frac{12}{5})^2 + (\frac{12}{5})^2 - 2(\frac{12}{5})^2 \cos \angle ACB = \frac{2016}{400} \rightarrow \overline{MN} = \sqrt{\frac{2016}{400}} = \boxed{\frac{3\sqrt{14}}{5}}$$

14. Chords $\overline{AB}$ and $\overline{CD}$ of a circle are perpendicular and intersect at a point P. If $\overline{AP} = 6$, $\overline{BP} = 12$, and $\overline{CD} = 22$, find the area of the circle.



Solution: Let $\overline{CP} = l$ and $\overline{PD} = r$. By Power of a Point on P, we know that $lr = 6 \cdot 12 = 72$. We also know that $l + r = 22$. Either by guessing and checking or noticing that l and r are the roots of the quadratic $x^2 - 22x + 72 = 0$ by Vieta's equations, we can determine that $l = 4$ and $r = 18$. Let the center of the circle be O, and let the feet of the altitudes from O to $\overline{CD}$ and $\overline{AB}$ be F and E, respectively. We know that F must be the midpoint of $\overline{CD}$, so we must have $\overline{CF} = \frac{22}{2} = 11$. However, we know that $\overline{CP} = 4$, so we know that $\overline{PF} = \overline{OE} = 11 - 4 = 7$. We also know that E is the midpoint of $\overline{AB}$, so we know that $\overline{EB} = \frac{6+12}{2} = 9$. It follows by the Pythagorean Theorem that $\overline{OB} = \sqrt{7^2 + 9^2} = \sqrt{130}$. Because this is a radius, it follows that the area of the circle is $(\sqrt{130})^2\pi = \boxed{130\pi}$ as desired.

15. Let $\triangle ABC$ be a right triangle with right angle $\angle C$. Let I be the incenter of $\triangle ABC$, and let M lie on $\overline{AC}$ and N on $\overline{BC}$, respectively, such that M, I, N are collinear and $\overline{MN}$ is parallel to $\overline{AB}$. If $\overline{AB} = 36$ and the perimeter of $\triangle CMN$ is 48, find the area of $\triangle ABC$.



Solution: Notice that because $\overline{NM}$ is parallel to $\overline{BA}$, we must have $\angle NIB \cong \angle IBA$. However, because I lies on the angle bisector of $\angle CBA$, it follows that $\angle IBA \cong CBI$. It follows that $\angle NIB \cong IBN$, and from here we can find that $\triangle NIB$ is an isosceles triangle with $\overline{NI} \cong \overline{NB}$. Similarly, we can find that $\triangle IMA$ is isosceles with $\overline{IM} \cong \overline{MA}$. It follows that the perimeter of $\triangle NMC$ is

$$48 = \overline{NC} + \overline{NM} + \overline{CM} = \overline{NC} + \overline{MC} + \overline{NI} + \overline{IM} = \overline{NC} + \overline{MC} + \overline{BN} + \overline{MA} = \overline{BC} + \overline{AC}$$

If we let $\overline{BC} = a$ and $\overline{AC} = b$, then it follows that $a + b = 48$ and $a^2 + b^2 = 36^2 = 1296$ by the Pythagorean Theorem. Squaring the first equation gives us $(a + b)^2 = a^2 + 2ab + b^2 = 2304$. Subtracting the second equation gives us $2ab = 2304 - 1296 = 1008$. Notice that the area of $\triangle ABC$ is $\frac{ab}{2}$, so it follows that our answer is

$$1008 \cdot \frac{\frac{ab}{2}}{2ab} = \frac{1008}{4} = \boxed{252}$$

3 Sources

1. 2009 November Harvard MIT Math Tournament General Problem 8
2. 2010 November Harvard MIT Math Tournament General Problem 3
3. 2010 November Harvard MIT Math Tournament General Problem 8
4. 2011 November Harvard MIT Math Tournament General Problem 2
5. 2012 November Harvard MIT Math Tournament General Problem 2
6. 2012 November Harvard MIT Math Tournament General Problem 6
7. 2013 November Harvard MIT Math Tournament General Problem 2
8. 2013 November Harvard MIT Math Tournament General Problem 5
9. 2013 November Harvard MIT Math Tournament General Problem 9
10. 2014 November Harvard MIT Math Tournament General Problem 1
11. 2014 November Harvard MIT Math Tournament General Problem 2
12. 2014 November Harvard MIT Math Tournament General Problem 4
13. 2014 November Harvard MIT Math Tournament General Problem 6
14. 2015 November Harvard MIT Math Tournament General Problem 4
15. 2015 November Harvard MIT Math Tournament General Problem 7