Geometry Handout 3

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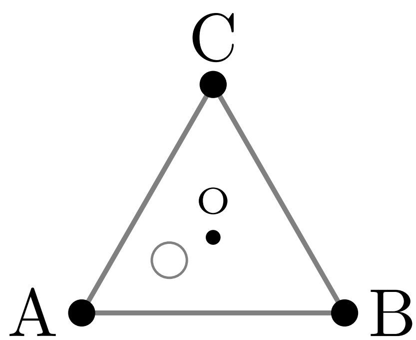
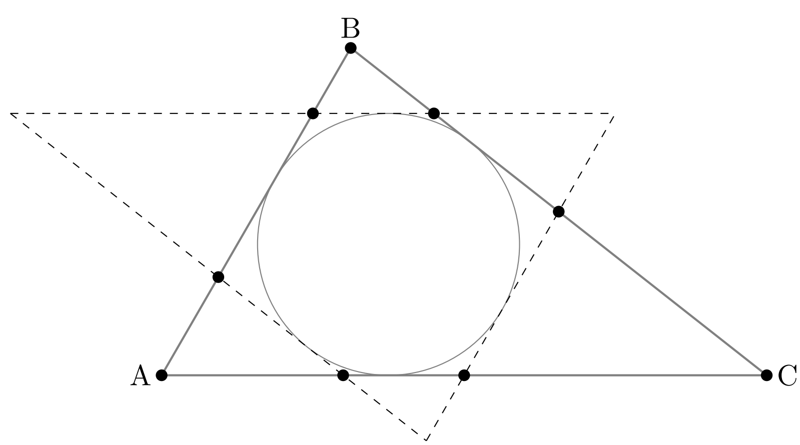
1 Problems

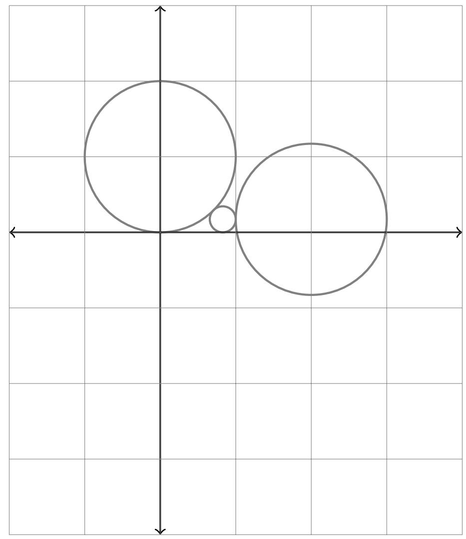
1. Let triangle $\triangle ABC$ be an equilateral triangle with height 13, and let O be its center. Point X is chosen at random from all points inside triangle $\triangle ABC$. Given that the circle of radius 1 centered at X lies entirely inside triangle $\triangle ABC$, what is the probability that the circle contains O?



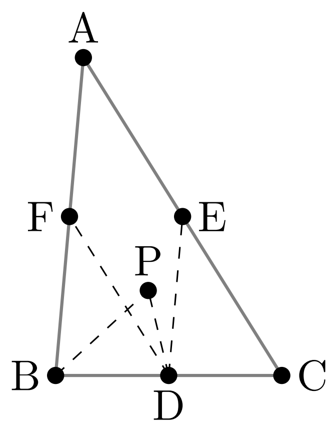
2. Triangle $\triangle ABC$ has $\overline{AB} = 5$, $\overline{BC} = 7$, and $\overline{CA} = 8$. New lines not containing but parallel to $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are drawn tangent to the incircle of $\triangle ABC$. What is the area of the hexagon formed by the sides of the original triangle and the newly drawn lines?



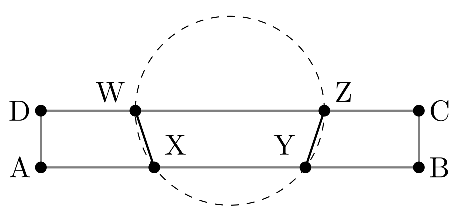
3. Two circles with radius one are drawn in the coordinate plane, one with center $(0, 1)$ and the other with center $(2, y)$ for some real number y between 0 and 1. A third circle is drawn so as to be tangent to both of the other two circles as well as the x-axis. What is the smallest possible radius for this third circle?



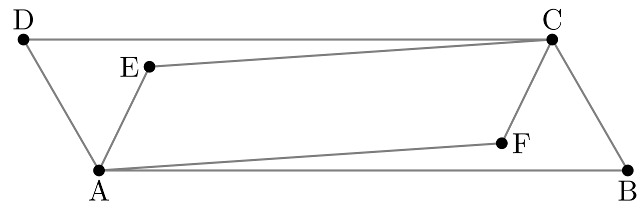
4. Let $\triangle ABC$ be a triangle, and let D, E, and F be the midpoints of sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Let the angle bisectors of $\angle FDE$ and $\angle FBD$ meet at P.Given that $\angle BAC = 37°$ and $\angle CBA = 85°$, determine the degree measure of $\angle BPD$.



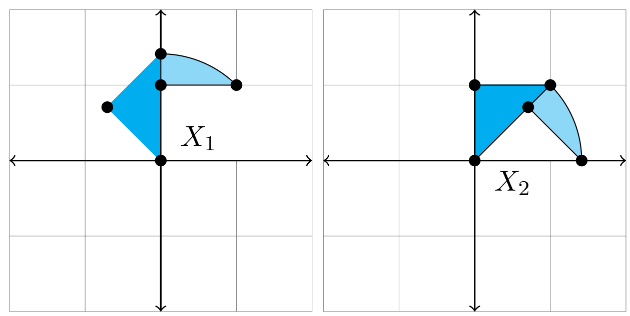
5. $ABCD$ is a rectangle with $\overline{AB} = 20$ and $\overline{BC} = 3$. A circle with radius 5, centered at the midpoint of $\overline{DC}$, meets the rectangle at four points: W, X, Y, and Z. Find the area of quadrilateral $WXYZ$.



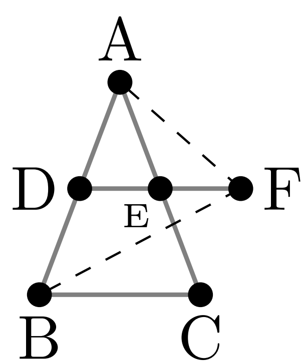
6. $ABCD$ is a parallelogram satisfying $\overline{AB} = 7$, $\overline{BC} = 2$, and $\angle DAB = 120°$. Parallelogram $ECFA$ is contained in $ABCD$ and is similar to it. Find the ratio of the area of $ECFA$ to the area of $ABCD$.



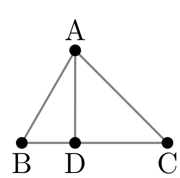
7. Plot points A, B, C at coordinates $(0,0)$, $(0,1)$, and $(1,1)$ in the plane, respectively. Let S denote the union of the two line segments $\overline{AB}$ and $\overline{BC}$. Let X_1 be the area swept out when Bobby rotates S *counterclockwise* $45°$ about point A. Let X_2 be the area swept out when Calvin rotates S *clockwise* $45°$ about point A. Find $\frac{X_1+X_2}{2}$.



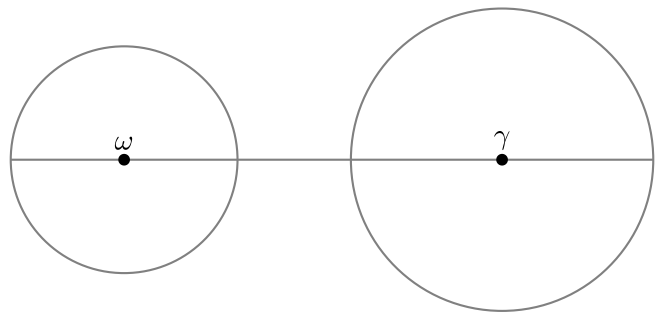
8. Let $\triangle ABC$ be an isosceles triangle with $\overline{AB} = \overline{AC}$. Let D and E be the midpoints of segments $\overline{AB}$ and $\overline{AC}$, respectively. Suppose that there exists a point F on ray $\vec{DE}$ outside of $\triangle ABC$ such that triangle $\triangle BFA$ is similar to triangle $\triangle ABC$. Compute $\frac{AB}{BC}$.



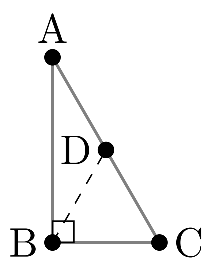
9. Let $\triangle ABC$ be a triangle and D a point on $\overline{BC}$ such that $\overline{AB} = \sqrt{2}$, $\overline{AC} = \sqrt{3}$, $\angle BAD = 30°$, and $\angle CAD = 45°$. Find $\overline{AD}$.



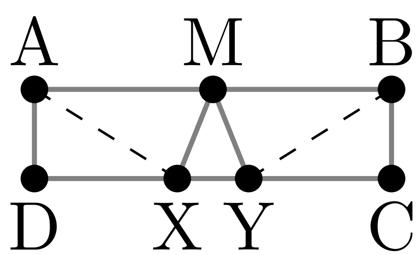
10. Two circles ω and γ have radii 3 and 4 respectively, and their centers are 10 units apart. Let x be the shortest possible distance between a point on ω and a point on γ, and let y be the longest possible distance between a point on ω and a point on γ. Find the product xy.



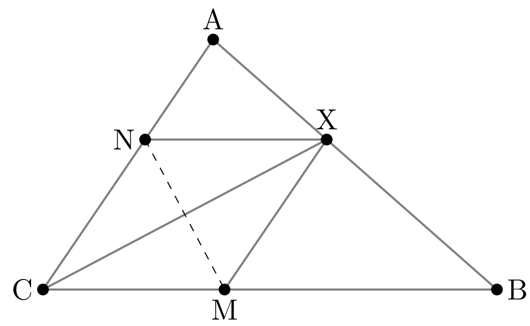
11. Let $\triangle ABC$ be a triangle with $\angle B = 90°$. Given that there exists a point D on $\overline{AC}$ such that $\overline{AD} = \overline{DC}$ and $\overline{BD} = \overline{BC}$, compute the value of the ratio $\frac{AB}{BC}$.



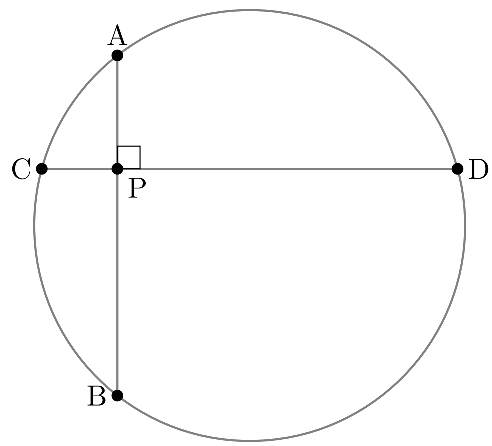
12. In rectangle $ABCD$ with area 1, point M is selected on $\overline{AB}$ and points X, Y are selected on $\overline{CD}$ such that $\overline{AX} < \overline{AY}$. Suppose that $\overline{AM} = \overline{BM}$. Given that the area of triangle $\triangle MXY$ is $\frac{1}{2014}$, compute the area of trapezoid $AXYB$. (Note: This diagram is not drawn to scale.)



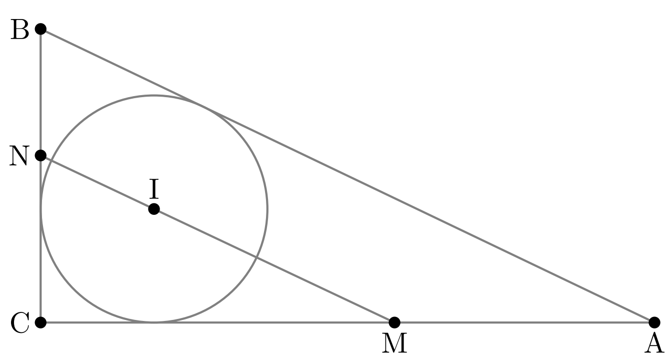
13. Let $\triangle ABC$ be a triangle with $\overline{AB} = 5$, $\overline{AC} = 4$, $\overline{BC} = 6$. The angle bisector of C intersects side $\overline{AB}$ at X. Points M and N are drawn on sides $\overline{BC}$ and $\overline{AC}$, respectively, such that $\overline{XM} \| \overline{AC}$ and $\overline{XN} \| \overline{BC}$. Compute the length $\overline{MN}$.



14. Chords $\overline{AB}$ and $\overline{CD}$ of a circle are perpendicular and intersect at a point P. If $\overline{AP} = 6$, $\overline{BP} = 12$, and $\overline{CD} = 22$, find the area of the circle.



15. Let $\triangle ABC$ be a right triangle with right angle $\angle C$. Let I be the incenter of $\triangle ABC$, and let M lie on $\overline{AC}$ and N on $\overline{BC}$, respectively, such that M, I, N are collinear and $\overline{MN}$ is parallel to $\overline{AB}$. If $\overline{AB} = 36$ and the perimeter of $\triangle CMN$ is 48, find the area of $\triangle ABC$.



2 Sources

1. 2009 November Harvard MIT Math Tournament General Problem 8
2. 2010 November Harvard MIT Math Tournament General Problem 3

3. 2010 November Harvard MIT Math Tournament General Problem 8
4. 2011 November Harvard MIT Math Tournament General Problem 2
5. 2012 November Harvard MIT Math Tournament General Problem 3
6. 2012 November Harvard MIT Math Tournament General Problem 6
7. 2013 November Harvard MIT Math Tournament General Problem 2
8. 2013 November Harvard MIT Math Tournament General Problem 5
9. 2013 November Harvard MIT Math Tournament General Problem 9
10. 2014 November Harvard MIT Math Tournament General Problem 1
11. 2014 November Harvard MIT Math Tournament General Problem 2
12. 2014 November Harvard MIT Math Tournament General Problem 4
13. 2014 November Harvard MIT Math Tournament General Problem 6
14. 2015 November Harvard MIT Math Tournament General Problem 4
15. 2015 November Harvard MIT Math Tournament General Problem 7