Geometry Handout #6

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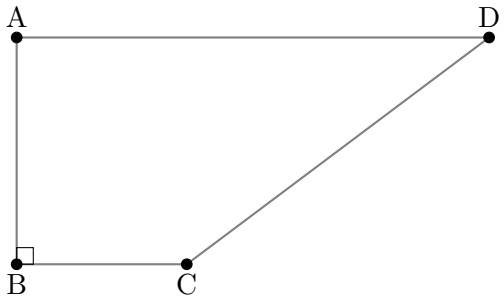
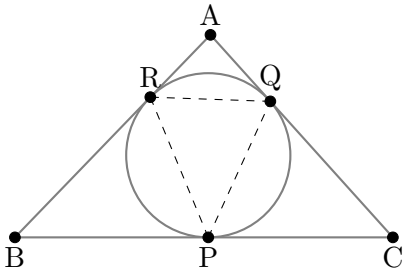
1 Problems

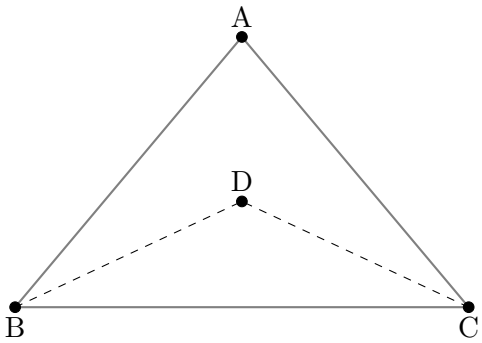
1. A quadrilateral $ABCD$ has a right angle at $\angle ABC$ and satisfies $\overline{AB} = 12$, $\overline{BC} = 9$, $\overline{CD} = 20$, and $\overline{DA} = 25$. Determine $\overline{BD}^2$.



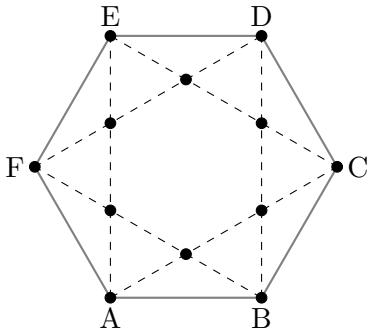
2. In $\triangle ABC$, $m\angle B = 46°$ and $m\angle C = 48°$. A circle is inscribed in $\triangle ABC$ and the points of tangency are connected to form $\triangle PQR$. What is the measure of the largest angle in $\triangle PQR$?



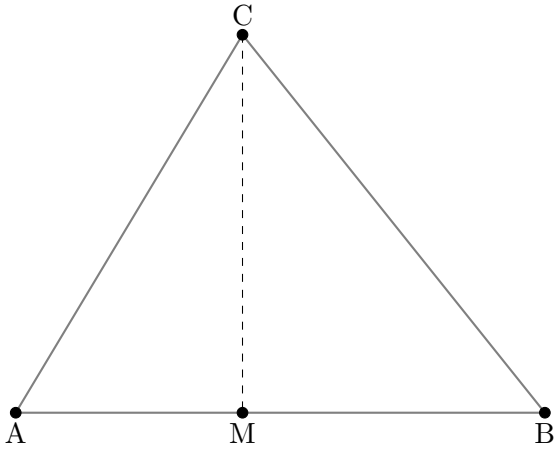
3. Let $\triangle ABC$ be a triangle. The angle bisectors of $\angle ABC$ and $\angle ACB$ intersect at D. If $\angle BAC = 80°$, what are all possible values for $\angle BDC$?



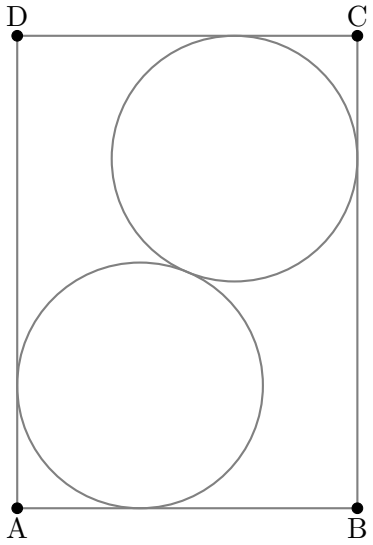
4. $ABCDEF$ is a regular hexagon. Let R be the overlap between $\triangle ACE$ and $\triangle BDF$. What is the area of R divided by the area of $ABCDEF$?



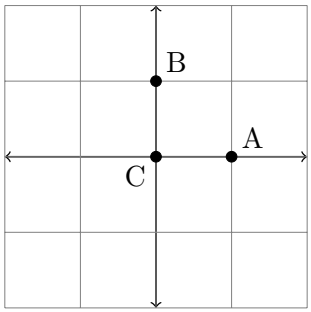
5. Let M be on segment BC of $\triangle ABC$ so that $\overline{AM} = 3$, $\overline{BM} = 4$, and $\overline{CM} = 5$. Find the largest possible area of $\triangle ABC$.



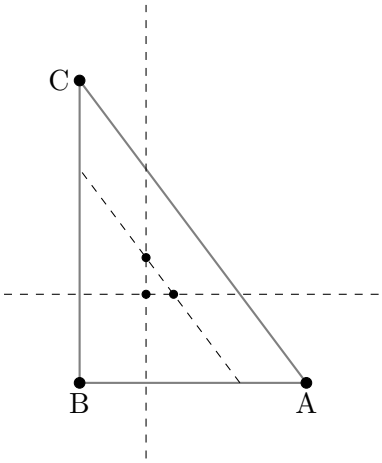
6. Let $ABCD$ be a rectangle. Circles C_1 and C_2 are externally tangent to each other. Furthermore, C_1 is tangent to $\overline{AB}$ and $\overline{AD}$, and C_2 is tangent to $\overline{CB}$ and $\overline{CD}$. If $\overline{AB} = 18$ and $\overline{BC} = 25$, then find the sum of the radii of the circles.



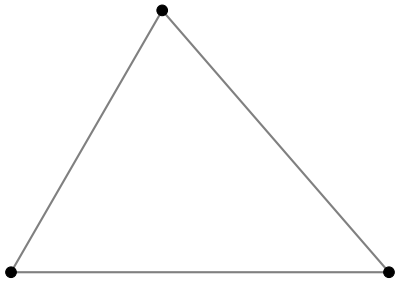
7. Let $A = (1, 0), B = (0, 1)$, and $C = (0, 0)$. There are three distinct points, P, Q, R, such that $\{A, B, C, P\}, \{A, B, C, Q\}, \{A, B, C, R\}$ are all parallelograms (vertices unordered). Find the area of $\triangle PQR$.



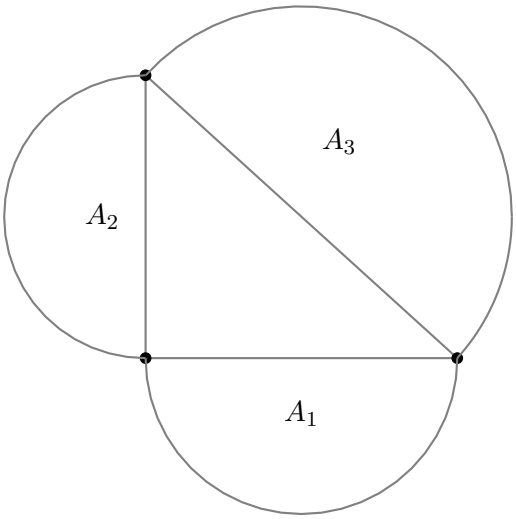
8. Triangle $\triangle ABC$ has side lengths $\overline{AB} = 3, \overline{BC} = 4$, and $\overline{CD} = 5$. Draw line l_A such that l_A is parallel to $\overline{BC}$ and splits the triangle into two polygons of equal area. Define lines l_B and l_C analogously. The intersection points of l_A, l_B, and l_C form a triangle. Determine its area.



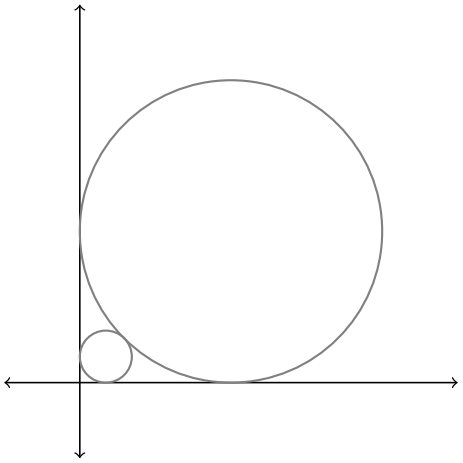
9. Suppose that two of the three sides of an acute triangle have lengths 20 and 16, respectively. How many possible integer values are there for the length of the third side?



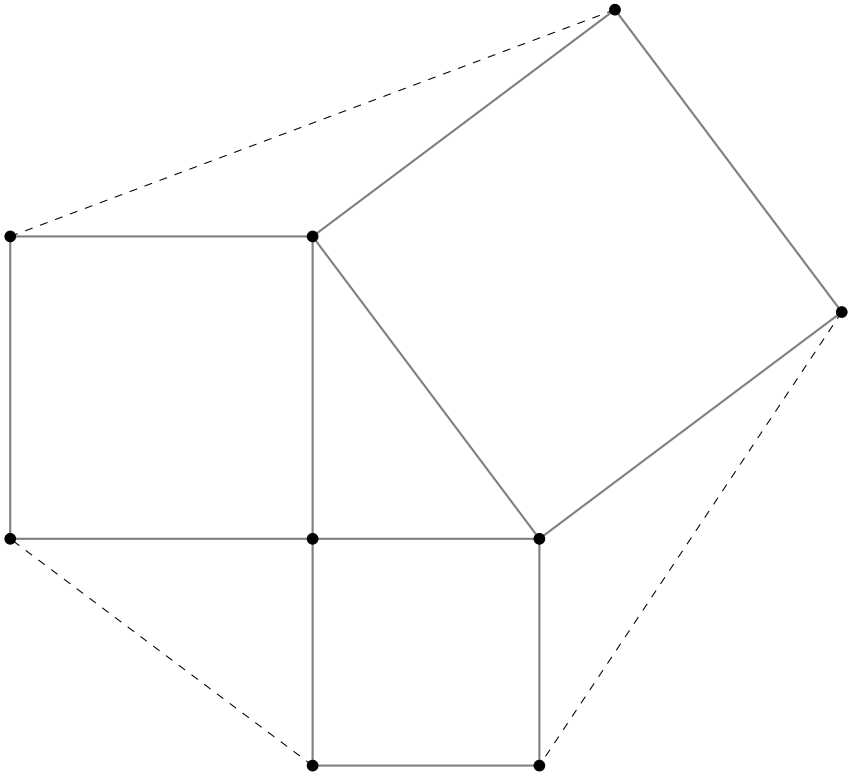
10. In the figure below, three semicircles are drawn outside the given right triangle. Given the areas $A_1 = 17$ and $A_2 = 14$, find the area A_3.



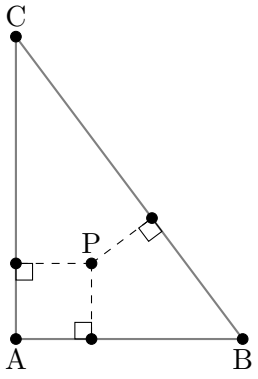
11. Consider a circle of radius 1 drawn tangent to the positive x and y axes. Now consider another smaller circle tangent to that circle and also tangent to the positive x and y axes. Find the radius of the smaller circle.



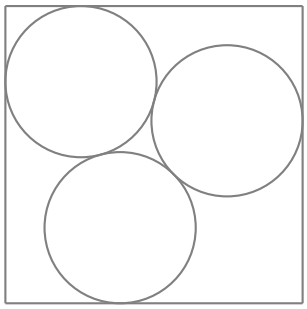
12. Suppose you have a triangle with side lengths $3, 4$, and 5. For each of the triangle's sides, draw a square on its outside. Connect the adjacent vertices in order, forming 3 new triangles (as in the diagram). What is the area of this convex region?



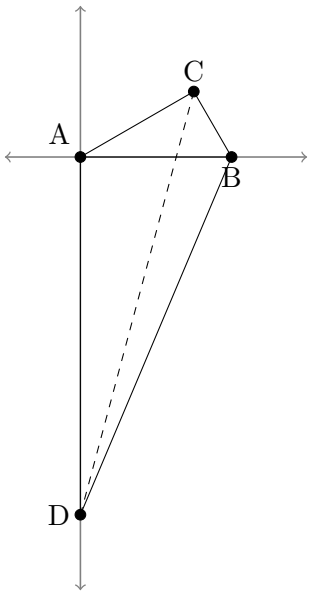
13. Let $\triangle ABC$ have side lengths $3, 4$, and 5. Let P be a point inside $\triangle ABC$. What is the minimum sum of the lengths of the altitudes from P to the side lengths of $\triangle ABC$?



14. Three circles of radius 1 are inscribed in a square of side length s such that the circles do not overlap or coincide with each other. What is the minimum s where such a configuration is possible?



15. Consider triangle $\triangle ABC$ in the xy-plane where A is at the origin, B lies on the positive x-axis, C is on the upper right quadrant, and $\angle A = 30°$, $\angle B = 60°$, and $\angle C = 90°$. Let the length $\overline{BC} = 1$. Draw the angle bisector l of angle $\angle C$, and let this intersect the y-axis at D. What is the area of quadrilateral $ADBC$?



2 Sources

1. 2015 Berkeley Math Tournament Spring Individual Problem 3
2. 2015 Berkeley Math Tournament Spring Individual Problem 7
3. 2015 Berkeley Math Tournament Spring Geometry Problem 1
4. 2015 Berkeley Math Tournament Spring Geometry Problem 2
5. 2015 Berkeley Math Tournament Spring Geometry Problem 3
6. 2015 Berkeley Math Tournament Spring Geometry Problem 4
7. 2015 Berkeley Math Tournament Spring Geometry Problem 5
8. 2015 Berkeley Math Tournament Spring Team Problem 4
9. 2016 Berkeley Math Tournament Fall Individual Problem 12
10. 2016 Berkeley Math Tournament Fall Team Problem 11
11. 2016 Berkeley Math Tournament Fall Team Problem 12
12. 2016 Berkeley Math Tournament Fall Team Problem 19

13. 2016 Berkeley Math Tournament Spring Individual Problem 4
14. 2016 Berkeley Math Tournament Spring Individual Problem 14
15. 2016 Berkeley Math Tournament Spring Individual Problem 17