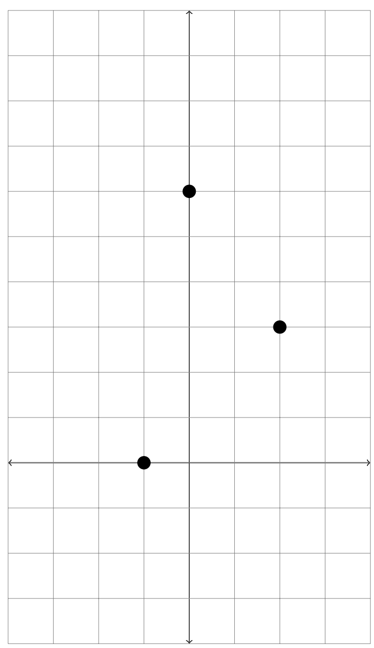
Geometry Handout #7

Walker Kroubalkian

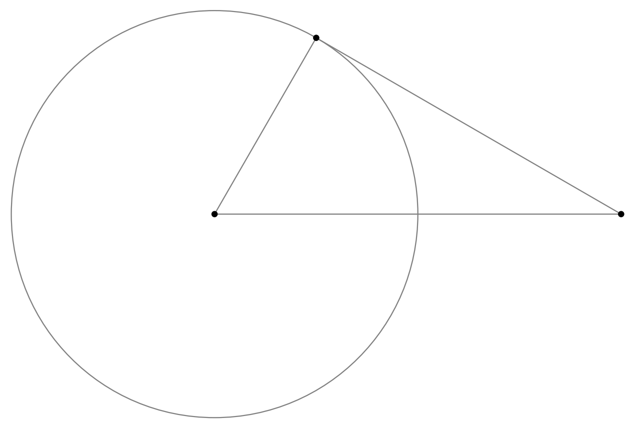
April 10, 2018

1 Problems

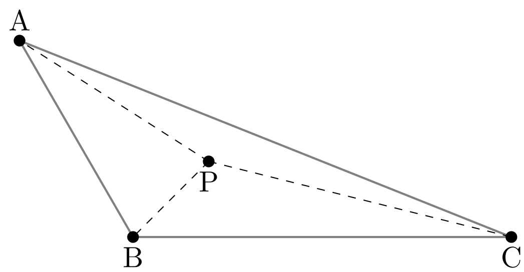
1. Let S be the set of points A in the Cartesian plane such that the four points A, $(2,3)$, $(-1,0)$, and $(0,6)$ form the vertices of a parallelogram. Let P be the convex polygon whose vertices are the points in S. What is the area of P?



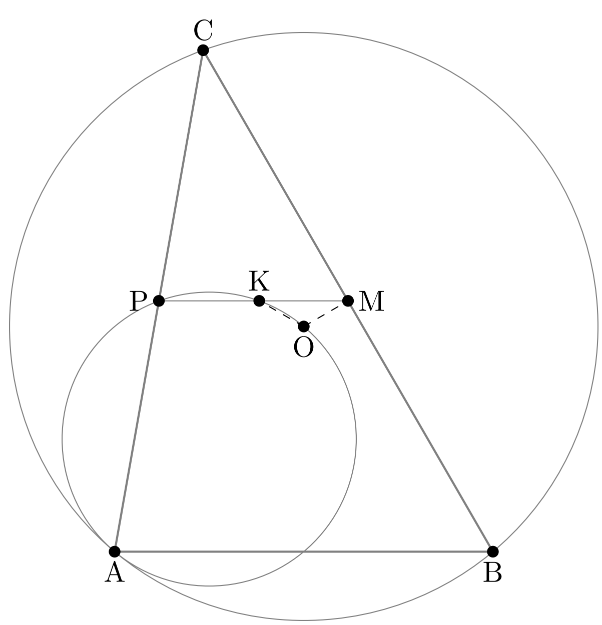
2. Two points are located 10 units apart, and a circle is drawn with radius r centered at one of the points. A tangent line to the circle is drawn from the other point. What value of r maximizes the area of the triangle formed by the two points and the point of tangency?



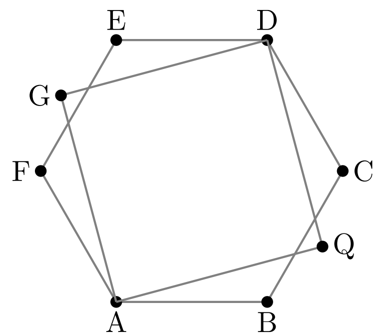
3. Let $\triangle ABC$ be a triangle with $\overline{AB} = 3$, $\overline{BC} = 5$, and $\overline{AC} = 7$, and let P be a point in its interior. If G_A, G_B, G_C are the centroids of $\triangle PBC, \triangle PAC, \triangle PAB$, respectively, find the maximum possible area of $\triangle G_A G_B G_C$.



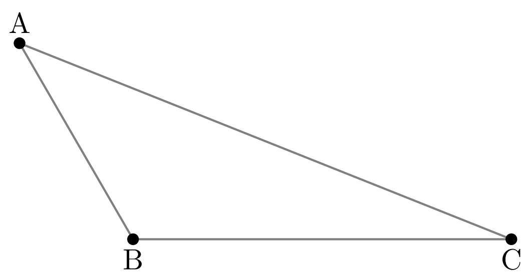
4. Triangle $\triangle ABC$ is drawn such that $\angle A = 80°$, $\angle B = 60°$, and $\angle C = 40°$. Let the circumcenter of $\triangle ABC$ be O, and let ω be the circle with diameter $\overline{AO}$. Circle ω intersects side $\overline{AC}$ at point P. Let M be the midpoint of side $\overline{BC}$, and let the intersection of ω and $\overline{PM}$ be K. Find the measure of $\angle MOK$.



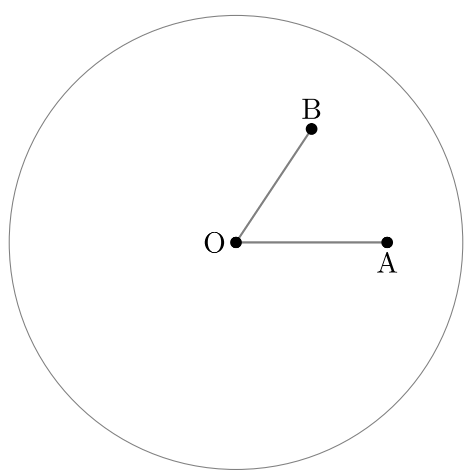
5. Let $ABCDEF$ be a regular hexagon with side length 1. Now construct square $AGDQ$. What is the area of the region inside the hexagon and not the square?



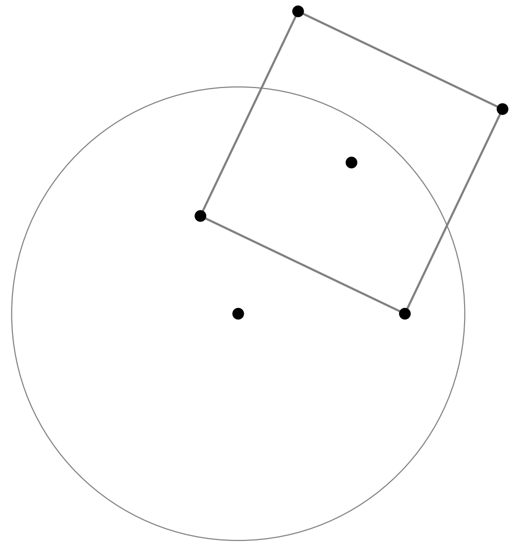
6. Suppose the side lengths of triangle $\triangle ABC$ are the roots of the polynomial $x^3 - 27x^2 + 222x - 540$. What is the product of its inradius and circumradius?



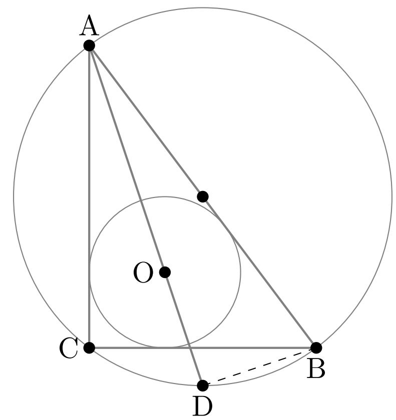
7. 2 darts are thrown randomly at a circular board with center O, such that each dart has an equal probability of hitting any point on the board. The points at which they land are marked A and B. What is the probability that $\angle AOB$ is acute?



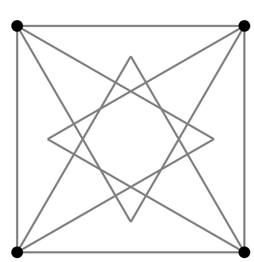
8. The center of a square of side length 1 is placed uniformly at random inside a circle of radius 1. Given that we are allowed to rotate the square about its center, what is the probability that the entire square is contained within the circle for some orientation of the square?



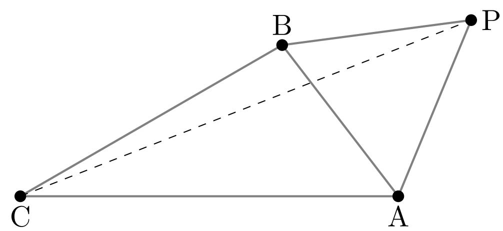
9. Let $\overline{AB} = 10$ be a diameter of circle P. Pick point C on the circle such that $\overline{AC} = 8$. Let the circle with center O be the incircle of $\triangle ABC$. Extend line $\overline{AO}$ to intersect circle P again at D. Find the length of $\overline{BD}$.



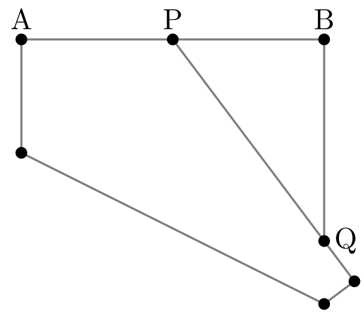
10. 4 equilateral triangles of side length 1 are drawn on the interior of a unit square, each one of which shares a side with one of the 4 sides of the unit square. What is the common area enclosed by all 4 equilateral triangles?



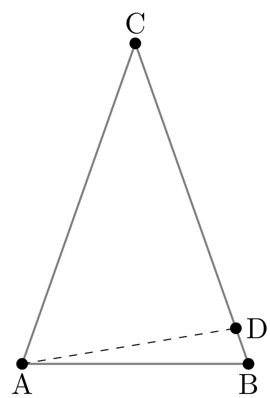
11. In triangle $\triangle ABC$, the angle at C is $30°$, side $\overline{BC}$ has length 4, and side $\overline{AC}$ has length 5. Let P be the point such that triangle $\triangle ABP$ is equilateral and non-overlapping with triangle $\triangle ABC$. Find the distance from C to P.



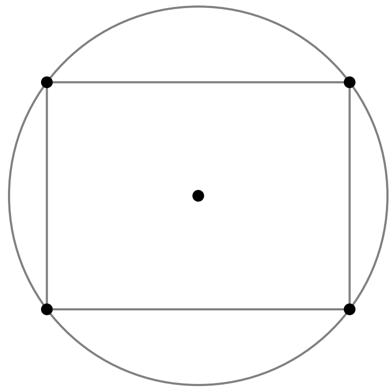
12. Take a square $ABCD$ of side length 1, and let P be the midpoint of $\overline{AB}$. Fold the square so that point D touches P, and let the intersection of the bottom edge $\overline{DC}$ with the right edge be Q. What is $\overline{BQ}$?



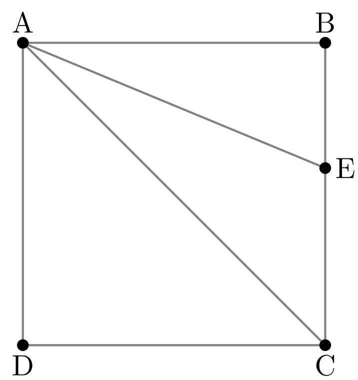
13. Let $\triangle ABC$ be a triangle with $\overline{AB} = 1$, $\overline{AC} = 3$, and $\overline{BC} = 3$. Let D be a point on $\overline{BC}$ such that $\overline{BD} = \frac{1}{3}$. What is the ratio of the area of $\triangle BAD$ to the area of $\triangle CAD$?



14. If a parallelogram with perimeter 14 and area 12 is inscribed in a circle, what is the radius of the circle?



15. Take a square $ABCD$ of side length 1, and draw $\overline{AC}$. Point E lies on $\overline{BC}$ such that $\overline{AE}$ bisects $\angle BAC$. What is the length of $\overline{BE}$?



2 Sources

1. 2017 Berkeley Math Tournament Spring Individual Problem 10
2. 2017 Berkeley Math Tournament Spring Individual Problem 13
3. 2017 Berkeley Math Tournament Spring Individual Problem 16
4. 2017 Berkeley Math Tournament Spring Individual Problem 17
5. 2017 Berkeley Math Tournament Spring Geometry Problem 3
6. 2017 Berkeley Math Tournament Spring Geometry Problem 5
7. 2017 Berkeley Math Tournament Spring Team Problem 4
8. 2017 Berkeley Math Tournament Spring Team Problem 6
9. 2017 Berkeley Math Tournament Spring Team Problem 9
10. 2017 Berkeley Math Tournament Spring Team Problem 13
11. 2017 Berkeley Math Tournament Spring Team Problem 15
12. 2017 Berkeley Math Tournament Fall Individual Problem 16
13. 2017 Berkeley Math Tournament Fall Team Problem 4
14. 2017 Berkeley Math Tournament Fall Team Problem 14
15. 2017 Berkeley Math Tournament Fall Team Problem 15