Geometry Handout 1

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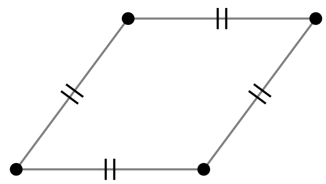
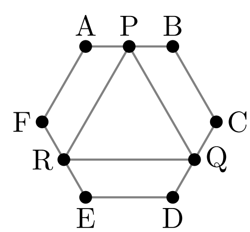
1 Problems

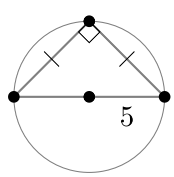
1. A rhombus has area 36 and the longer diagonal is twice as long as the shorter diagonal. What is the perimeter of the rhombus?



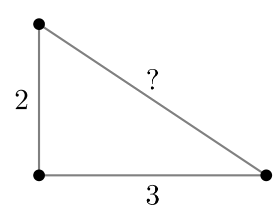
2. Given regular hexagon $ABCDEF$, compute the probability that a randomly chosen point inside the hexagon is inside triangle PQR, where P is the midpoint of $\overline{AB}$, Q is the midpoint of $\overline{CD}$, and R is the midpoint of $\overline{EF}$.



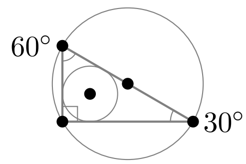
3. An isosceles right triangle is inscribed in a circle of radius 5, thereby separating the circle into four regions. Compute the sum of the areas of the two smallest regions.



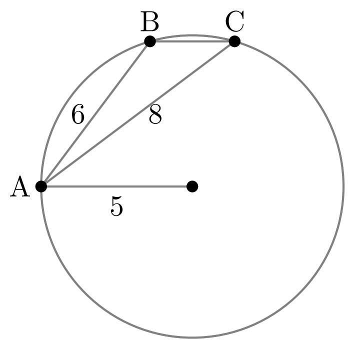
4. A triangle with side lengths 2 and 3 has an area of 3. Compute the third side length of the triangle.



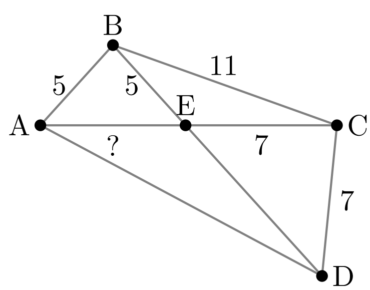
5. Compute the square of the distance between the incenter (center of the inscribed circle) and circumcenter (center of the circumscribed circle) of a 30-60-90 triangle with hypotenuse of length 2.



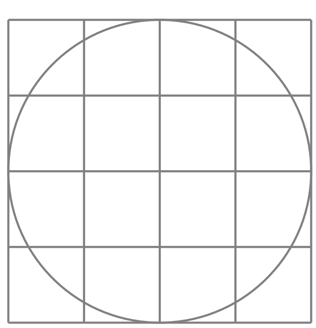
6. Points A, B, and C lie on a circle of radius 5 such that $\overline{AB} = 6$ and $\overline{AC} = 8$. Find the smaller of the two possible values of $\overline{BC}$.



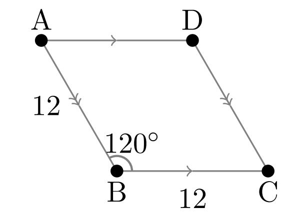
7. In quadrilateral $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. If $\overline{AB} \cong \overline{BE} = 5$, $\overline{EC} \cong \overline{CD} = 7$, and $\overline{BC} = 11$, compute $\overline{AE}$.



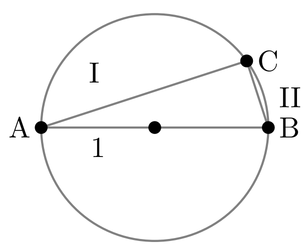
8. Robin has obtained a circular pizza with radius 2. However, being rebellious, instead of slicing the pizza radially, he decides to slice the pizza into 4 strips of equal width both vertically and horizontally. What is the area of the smallest piece of pizza?



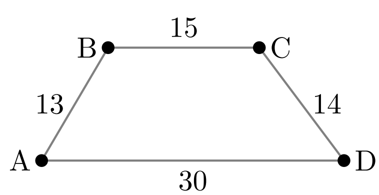
9. $ABCD$ is a parallelogram. $\overline{AB} = \overline{BC} = 12$, and $\angle ABC = 120°$. Calculate the area of parallelogram $ABCD$.



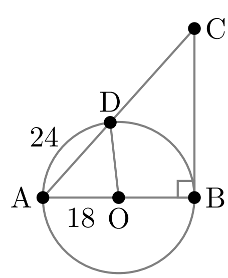
10. A circle with radius 1 has diameter $\overline{AB}$. C lies on this circle such that the ratio of arc $\overline{AC}$ to arc $\overline{BC}$ is 4. Segment $\overline{AC}$ divides the circle into two parts, and we will label the smaller part Region I. Similarly, segment $\overline{BC}$ also divides the circle into two parts, and we will denote the smaller one as Region II. Find the positive difference between the areas of Regions I and II.



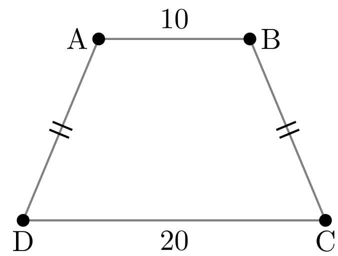
11. In trapezoid $ABCD$, $\overline{BC}$ is parallel to $\overline{AD}$, $\overline{AB} = 13$, $\overline{BC} = 15$, $\overline{CD} = 14$, and $\overline{AD} = 30$. Find the area of $ABCD$.



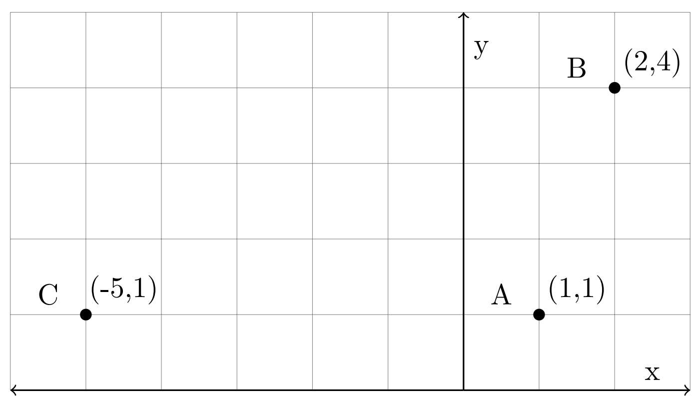
12. Circle O has radius 18. From diameter $\overline{AB}$, there exists a point C such that $\overline{BC}$ is tangent to O and $\overline{AC}$ intersects O at a point D, with $\overline{AD} = 24$. What is the length of $\overline{BC}$?



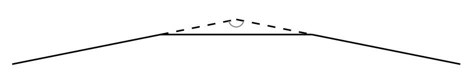
13. Isosceles trapezoid $ABCD$ has $\overline{AB} = 10$, $\overline{CD} = 20$, $\overline{BC} \cong \overline{AD}$, and an area of 180. Compute the length of $\overline{BC}$.



14. The coordinates of three vertices of a parallelogram are A $(1, 1)$, B $(2, 4)$, and C $(-5, 1)$. Compute the area of the parallelogram.



15. Given a regular 2014-gon, we construct 2014 isosceles triangles on the exterior of the polygon such that each isosceles triangle has an edge of the polygon as its base and has legs formed by the extensions of the two adjacent sides. Compute in degrees the largest angle of one such triangle. (Note: This figure is not drawn to scale.)



2 Sources

1. 2013 Stanford Math Tournament General Problem 5
2. 2013 Stanford Math Tournament General Problem 15
3. 2013 Stanford Math Tournament General Problem 17
4. 2013 Stanford Math Tournament General Problem 19
5. 2013 Stanford Math Tournament General Problem 24
6. 2013 Stanford Math Tournament Geometry Tiebreaker Problem 2
7. 2013 Stanford Math Tournament Geometry Tiebreaker Problem 3
8. 2013 Stanford Math Tournament Geometry Problem 3
9. 2012 Stanford Math Tournament General Problem 15
10. 2012 Stanford Math Tournament General Problem 17
11. 2012 Stanford Math Tournament General Problem 21
12. 2012 Stanford Math Tournament General Problem 23

13. 2014 Stanford Math Tournament General Problem 7
14. 2014 Stanford Math Tournament General Problem 10
15. 2014 Stanford Math Tournament General Problem 15