Geometry Handout #4

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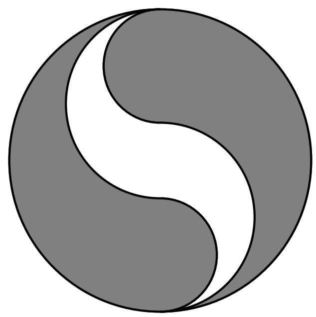
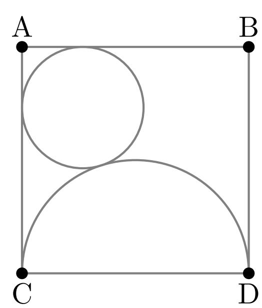
1 Problems

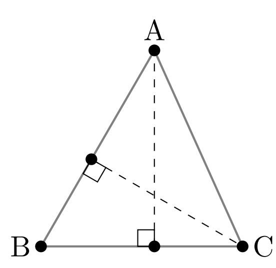
1. S-Corporation designs its logo by linking together 4 semicircles along the diameter of a unit circle. Find the perimeter of the shaded portion of the logo.



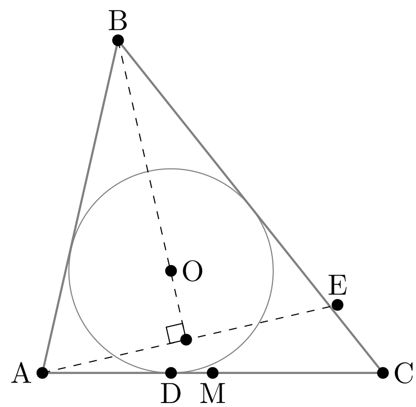
2. Let $ABCD$ be a square with side length 2, and let a semicircle with flat side CD be drawn inside the square. Of the remaining area inside the square outside the semi-circle, the largest circle is drawn. What is the radius of this circle?



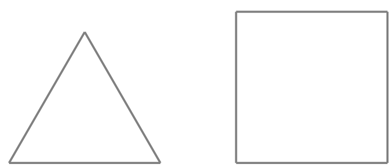
3. Triangle ABC satisfies the property that $\angle A = a\log x$, $\angle B = a\log 2x$, and $\angle C = a\log 4x$ radians, for some real numbers a and x. If the altitude to side $\overline{AB}$ has length 8 and the altitude to side $\overline{BC}$ has length 9, find the area of $\triangle ABC$.



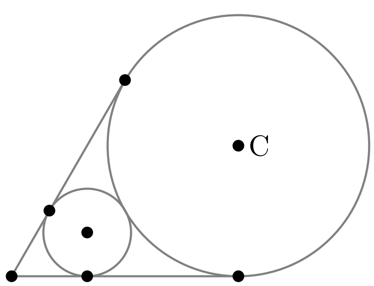
4. Triangle $\triangle ABC$ has incircle O that is tangent to $\overline{AC}$ at D. Let M be the midpoint of $\overline{AC}$. E lies on $\overline{BC}$ so that line $\overline{AE}$ is perpendicular to $\overline{BO}$ extended. If $\overline{AC} = 2013$, $\overline{AB} = 2014$, and $\overline{DM} = 249$, find $\overline{CE}$.



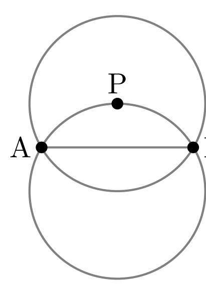
5. What is the area of a square whose sides are the same length as the sides of an equilateral triangle with area 4?



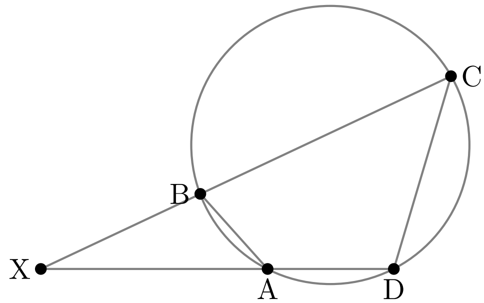
6. Two rays start from a common point and have an angle of 60 degrees. Circle C is drawn with radius 42 such that it is tangent to the two rays. Find the radius of the circle that has radius smaller than circle C and is also tangent to C and the two rays.



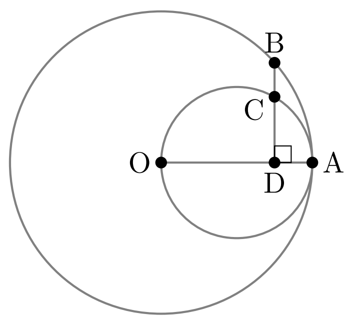
7. Points A and B are fixed points in the plane such that $\overline{AB} = 1$. Find the area of the region consisting of all points P such that $\angle APB > 120°$.



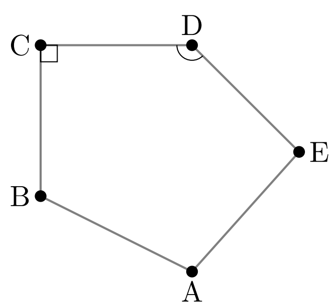
8. Let $ABCD$ be a cyclic quadrilateral where $\overline{AB} = 4$, $\overline{BC} = 11$, $\overline{CD} = 8$, and $\overline{DA} = 5$. If $\overline{BC}$ and $\overline{DA}$ intersect at X, find the area of $\triangle XAB$.



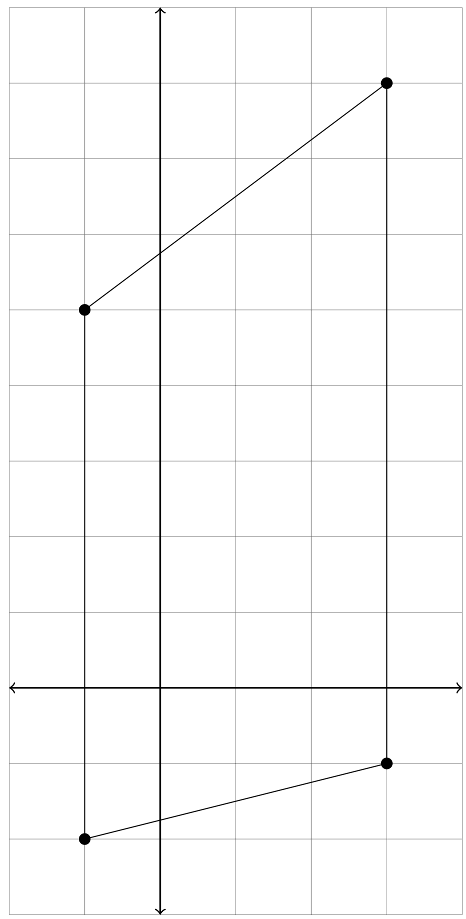
9. Circle C_1 has center O and radius $\overline{OA}$, and circle C_2 has diameter $\overline{OA}$. $\overline{AB}$ is a chord of circle C_1 and $\overline{BD}$ may be constructed with D on $\overline{OA}$ such that $\overline{BD}$ and $\overline{OA}$ are perpendicular. Let C be the point where C_2 and $\overline{BD}$ intersect. If $\overline{AC} = 1$, find $\overline{AB}$.



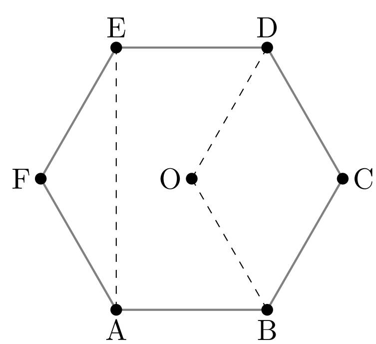
10. Given pentagon $ABCDE$ with $\overline{BC} \cong \overline{CD} \cong \overline{DE} = 4$, $\angle BCD = 90°$, and $\angle CDE = 135°$, what is the length of $\overline{BE}$?



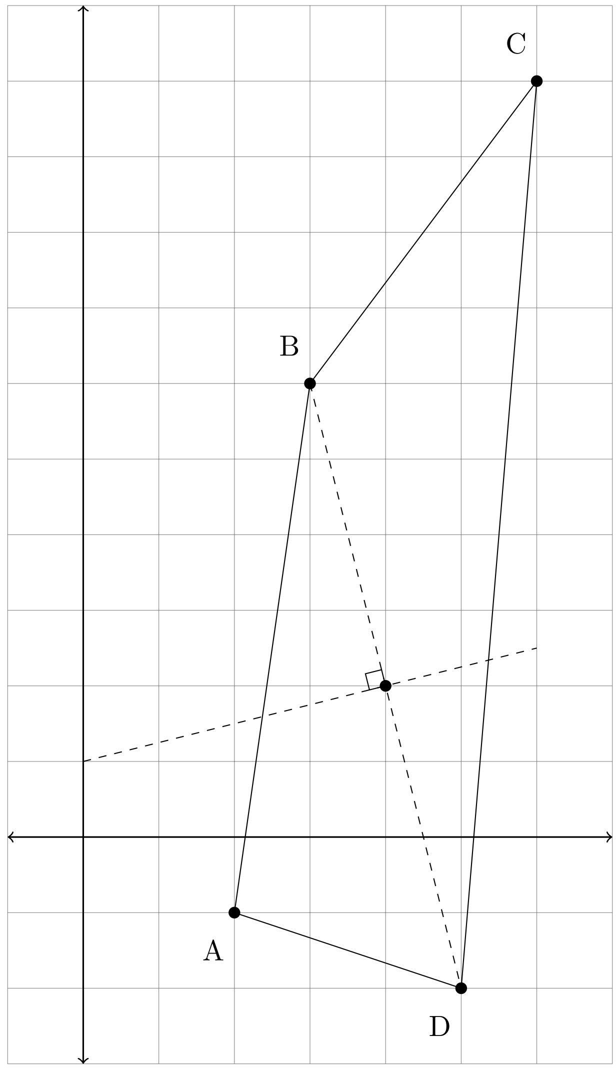
11. Find the area of the convex quadrilateral with vertices at the points $(-1, 5), (3, 8), (3, -1)$, and $(-1, -2)$.



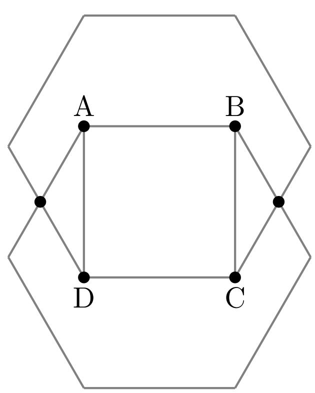
12. Given regular hexagon $ABCDEF$ with center O and side length 6, what is the area of pentagon $ABODE$?



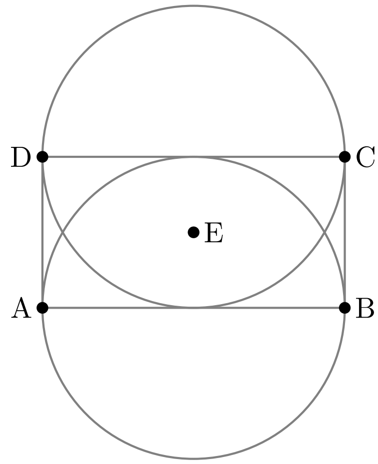
13. A quadrilateral $ABCD$ is defined by the points $A = (2, -1)$, $B = (3, 6)$, $C = (6, 10)$, and $D = (5, -2)$. Let l be the line that intersects and is perpendicular to the shorter diagonal at its midpoint. What is the slope of l?



14. Given square $ABCD$ with side length 3, we construct two regular hexagons on side $\overline{AB}$ and $\overline{CD}$ such that the hexagons contain the square. What is the area of the intersection of the two hexagons?



15. Let E be a random point inside rectangle $ABCD$ with side lengths $\overline{AB} = 2$ and $\overline{BC} = 1$. What is the probability that angles $\angle AEB$ and $\angle CED$ are both obtuse?



Note: Originally this problem asked for the probability that angles $\angle ABE$ and $\angle CDE$ are obtuse. However, because the probability of this event is obviously 0, I believe the problem writers actually meant for the question to be as stated above. This rewritten problem coincides with the answer on their answer key.

2 Sources

1. 2013 Berkeley Math Tournament Spring Individual Problem 2
2. 2013 Berkeley Math Tournament Spring Individual Problem 4
3. 2013 Berkeley Math Tournament Spring Individual Problem 12
4. 2013 Berkeley Math Tournament Spring Individual Problem 14
5. 2014 Berkeley Math Tournament Fall Speed Problem 32
6. 2013 Berkeley Math Tournament Spring Geometry Problem 2
7. 2013 Berkeley Math Tournament Spring Geometry Problem 5
8. 2013 Berkeley Math Tournament Spring Geometry Problem 6
9. 2013 Berkeley Math Tournament Spring Team Problem 5
10. 2014 Berkeley Math Tournament Fall Individual Problem 7
11. 2014 Berkeley Math Tournament Fall Individual Problem 9
12. 2014 Berkeley Math Tournament Fall Individual Problem 14
13. 2014 Berkeley Math Tournament Fall Individual Problem 18
14. 2014 Berkeley Math Tournament Fall Team Problem 7
15. 2014 Berkeley Math Tournament Fall Team Problem 14