Geometry Handout #5

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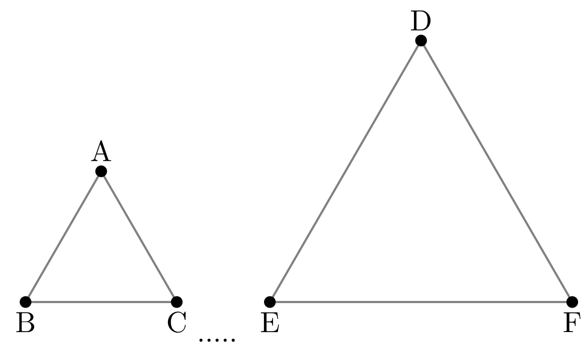
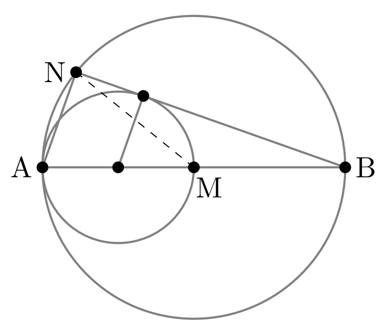
February 13, 2018

1 Problems

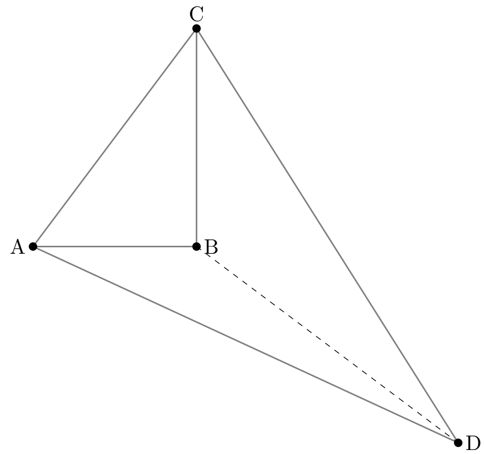
1. Suppose $\triangle ABC$ is similar to $\triangle DEF$, with A, B, and C corresponding to D, E, and F respectively. If $\overline{AB} = \overline{EF}$, $\overline{BC} = \overline{FD}$, and $\overline{CA} = \overline{DE} = 2$, determine the area of $\triangle ABC$.



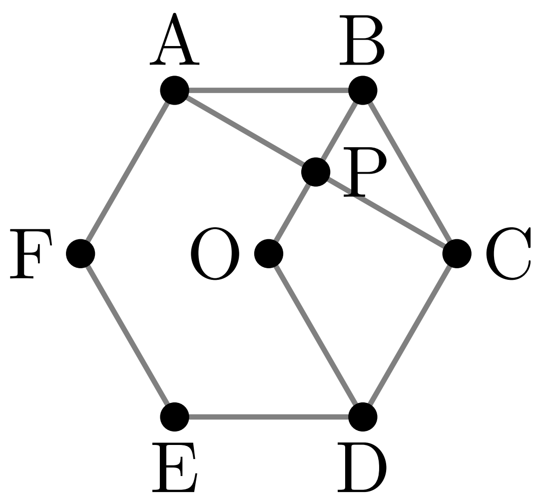
2. Line segment $\overline{AB}$ has length 4 and midpoint M. Let circle C_1 have diameter $\overline{AB}$, and let circle C_2 have diameter $\overline{AM}$. Suppose a tangent of circle C_2 goes through point B to intersect circle C_1 at N. Determine the area of triangle $\triangle AMN$.



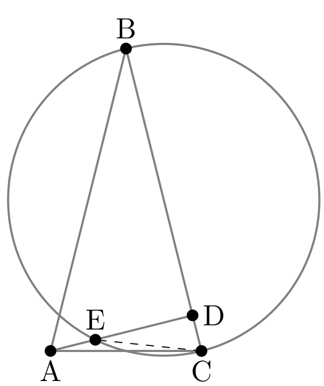
3. Suppose four coplanar points A, B, C, and D satisfy $\overline{AB} = 3$, $\overline{BC} = 4$, $\overline{CA} = 5$, and $\overline{BD} = 6$. Determine the maximal possible area of $\triangle ACD$.



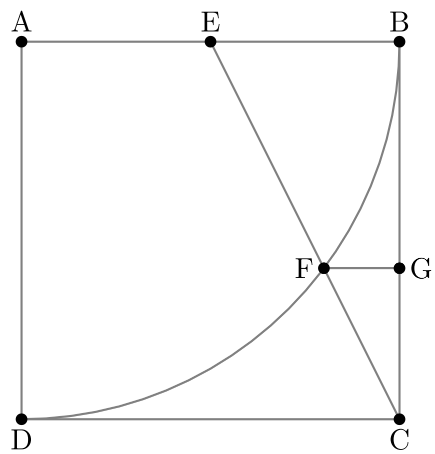
4. Regular hexagon $ABCDEF$ has side length 2 and center O. The point P is defined as the intersection of $\overline{AC}$ and $\overline{OB}$. Find the area of quadrilateral $OPCD$.



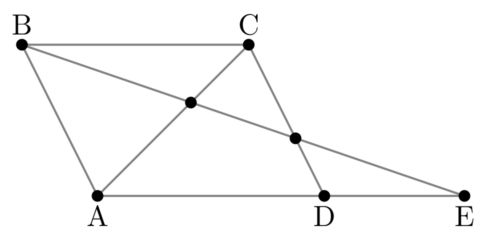
5. Consider an isosceles triangle $\triangle ABC$ ($\overline{AB} = \overline{BC}$). Let D be on $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$ and O be a circle with diameter $\overline{BC}$. Suppose that segment $\overline{AD}$ intersects circle O at E. If $\overline{CA} = 2$, what is $\overline{CE}$?



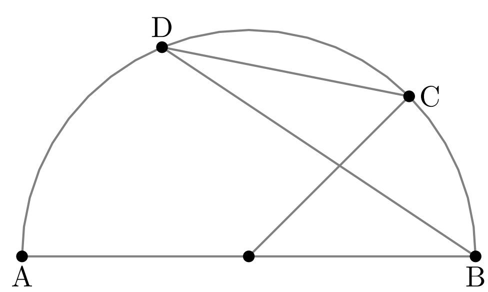
6. Square $ABCD$ has side length 5 and arc $\overset{\frown}{BD}$ with center A. E is the midpoint of $\overline{AB}$ and $\overline{CE}$ intersects arc $\overset{\frown}{BD}$ at F. G is placed onto $\overline{BC}$ such that $\overline{FG}$ is perpendicular to $\overline{BC}$. What is the length of $\overline{FG}$?



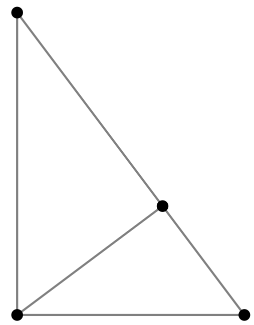
7. Consider a parallelogram $ABCD$. E is a point on ray $\vec{AD}$. $\overline{BE}$ intersects $\overline{AC}$ at F and $\overline{CD}$ at G. If $\overline{BF} = \overline{EG}$ and $\overline{BC} = 3$, find the length of $\overline{AE}$.



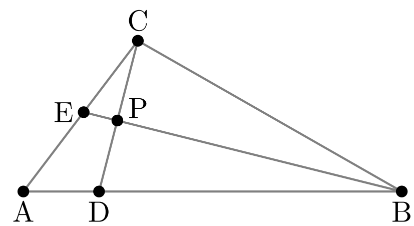
8. Semicircle O has diameter $\overline{AB} = 12$. Arc $\overset{\frown}{AC} = 135°$. Let D be the midpoint of arc $\overset{\frown}{AC}$. Compute the (area of the) region bounded by the lines $\overline{CD}$ and $\overline{DB}$ and the arc $\overset{\frown}{CB}$.



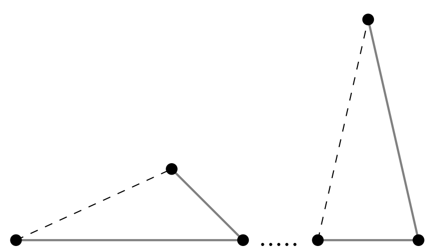
9. In a right triangle, the altitude from a vertex to the hypotenuse splits the hypotenuse into two segments of lengths a and b. If the right triangle has area T and is inscribed in a circle of area C, find ab in terms of T and C.



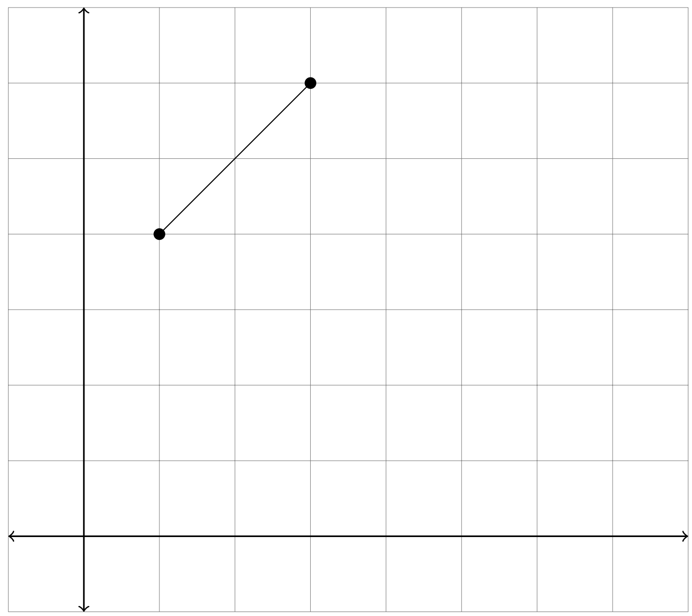
10. Let $\triangle ABC$ be a triangle with $\overline{AB} = 16$, $\overline{AC} = 10$, $\overline{BC} = 18$. Let D be a point on $\overline{AB}$ such that $4\overline{AD} = \overline{AB}$ and let E be the foot of the angle bisector from B onto $\overline{AC}$. Let P be the intersection of $\overline{CD}$ and $\overline{BE}$. Find the area of the quadrilateral $ADPE$.



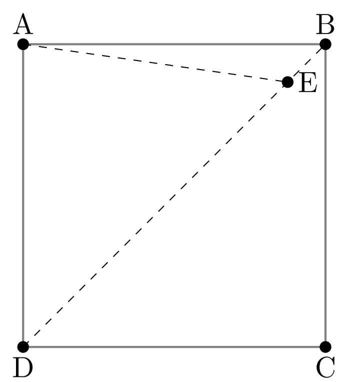
11. Two sides of an isosceles triangle $\triangle ABC$ have lengths 9 and 4. What is the area of $\triangle ABC$?



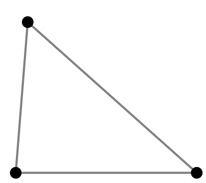
12. An isosceles triangle has two vertices at $(1, 4)$ and $(3, 6)$. Find the x-coordinate of the third vertex assuming it lies on the x-axis.



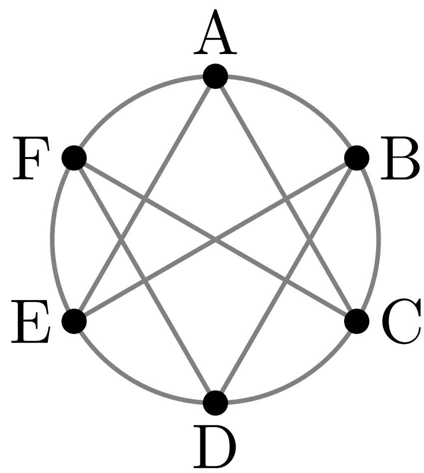
13. A point P is inside the square $ABCD$. IF $\overline{PA} = 5$, $\overline{PB} = 1$, and $\overline{PD} = 7$, then what is $\overline{PC}$?



14. Two sides of a triangle have lengths 20 and 30. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?



15. Assume the A, B, C, D, E, and F are equally spaced on a circle of radius 1, as in the figure below. Find the area of the kite bounded by the lines $\overline{EA}$, $\overline{AC}$, $\overline{FC}$, and $\overline{BE}$.



2 Sources

1. 2014 Berkeley Math Tournament Spring Individual Problem 2
2. 2014 Berkeley Math Tournament Spring Individual Problem 8
3. 2014 Berkeley Math Tournament Spring Individual Problem 12
4. 2014 Berkeley Math Tournament Spring Geometry Problem 2
5. 2014 Berkeley Math Tournament Spring Geometry Problem 3
6. 2014 Berkeley Math Tournament Spring Geometry Problem 6
7. 2014 Berkeley Math Tournament Spring Geometry Problem 7
8. 2014 Berkeley Math Tournament Spring Geometry Problem 8
9. 2014 Berkeley Math Tournament Spring Team Problem 4
10. 2014 Berkeley Math Tournament Spring Team Problem 13
11. 2015 Berkeley Math Tournament Fall Individual Problem 3
12. 2015 Berkeley Math Tournament Fall Individual Problem 9
13. 2015 Berkeley Math Tournament Fall Individual Problem 13
14. 2015 Berkeley Math Tournament Fall Individual Problem 16
15. 2015 Berkeley Math Tournament Fall Individual Problem 18