Geometry Handout #8

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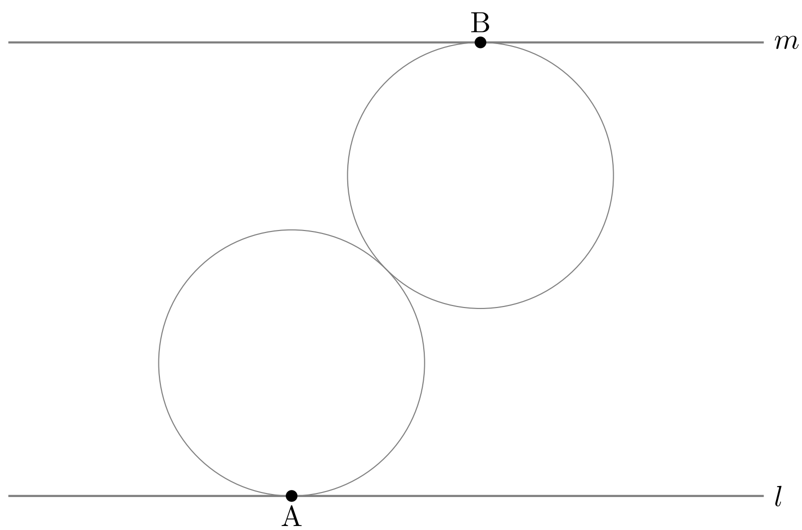
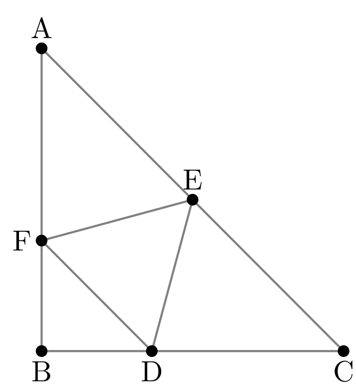
1 Problems

1. The figure below shows two parallel lines, l and m, that are a distance 12 apart:



A circle is tangent to line l at point A. Another circle is tangent to line m at point B. The two circles are congruent and tangent to each other as shown. The distance between A and B is 13. What is the radius of each circle? Express your answer as a fraction in reduced form.

2. The figure below shows a right triangle $\triangle ABC$.

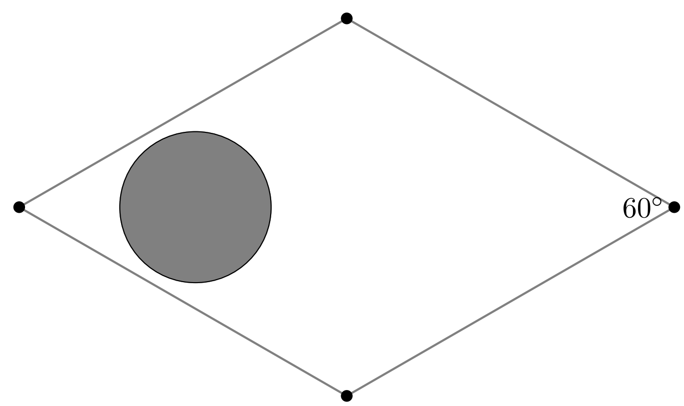


The legs $\overline{AB}$ and $\overline{BC}$ each have length 4. An equilateral triangle $\triangle DEF$ is inscribed in $\triangle ABC$ as shown. Point D is the midpoint of $\overline{BC}$. What is the area of $\triangle DEF$? Express your answer in the form $m\sqrt{3} - n$, where m and n are positive integers.

3. The bases of a trapezoid have lengths 10 and 21, and the legs have lengths $\sqrt{34}$ and $3\sqrt{5}$. What is the area of the trapezoid? Express your answer as a fraction in simplest form.

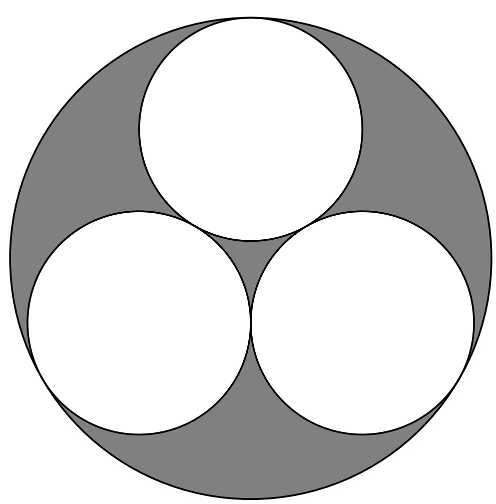
4. The triangle $\triangle ABC$ lies on the coordinate plane. The midpoint of $\overline{AB}$ has coordinates $(-16, -63)$, the midpoint of $\overline{AC}$ has coordinates $(13, 50)$, and the midpoint of $\overline{BC}$ has coordinates $(6, -85)$. What are the coordinates of point A? Express your answer as an ordered pair (x, y).

5. In the figure below, each side of the rhombus has length 5 centimeters.



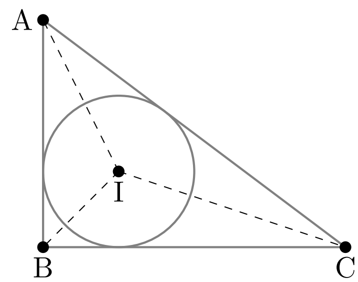
The circle lies entirely within the rhombus. The area of the circle is n square centimeters, where n is a positive integer. Compute the number of possible values of n.

6. In the figure below, the three small circles are congruent and tangent to each other. The large circle is tangent to the three small circles.

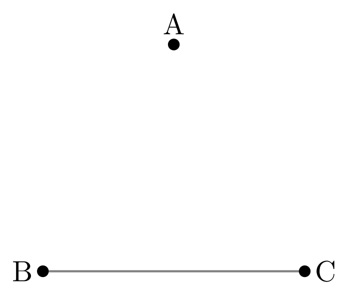


The area of the large circle is 1. What is the area of the shaded region? Express your answer in the form $a\sqrt{n}-b$, where a and b are positive integers and n is a square-free positive integer.

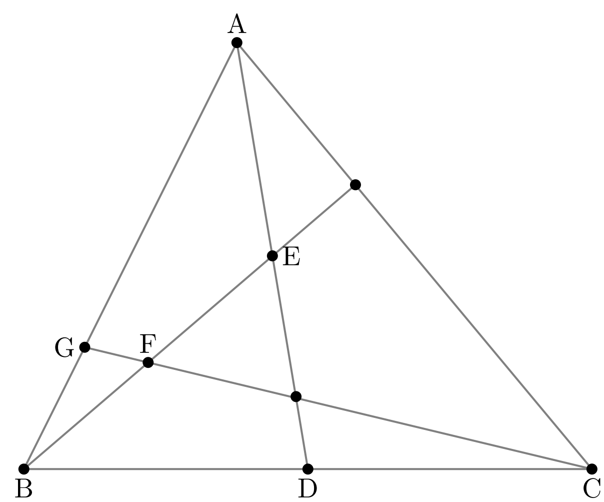
7. Let $\triangle ABC$ be a triangle with $\overline{AB}=3$, $\overline{BC}=4$, and $\overline{AC}=5$. Let I be the center of the circle inscribed in $\triangle ABC$. What is the product of $\overline{AI}, \overline{BI}$, and $\overline{CI}$?



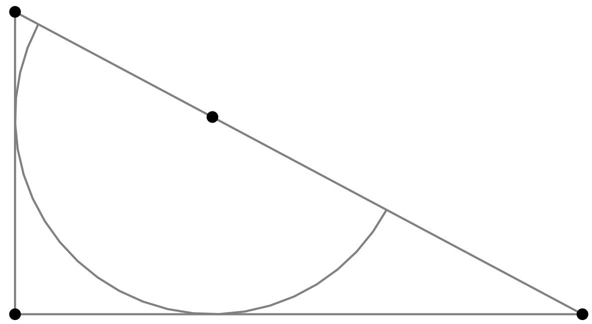
8. In the figure below, points A, B, and C are a distance 6 from each other. Say that a point X is *reachable* if there is a path (not necessarily) straight connecting A and X of length at most 8 that does not intersect the interior of $\overline{BC}$. (Both X and the path must lie on the plane containing A, B, and C.) Let R be the set of *reachable* points. What is the area of R? Express your answer in the form $m\pi + n\sqrt{3}$ where m and n are integers.



9. Let $\triangle ABC$ be a triangle. Let D be the midpoint of $\overline{BC}$, let E be the midpoint of $\overline{AD}$, and let F be the midpoint of $\overline{BE}$. Let G be the point where the lines $\overline{AB}$ and $\overline{CF}$ intersect. What is the value of $\frac{\overline{AG}}{\overline{AB}}$? Express your answer as a fraction in simplest form.

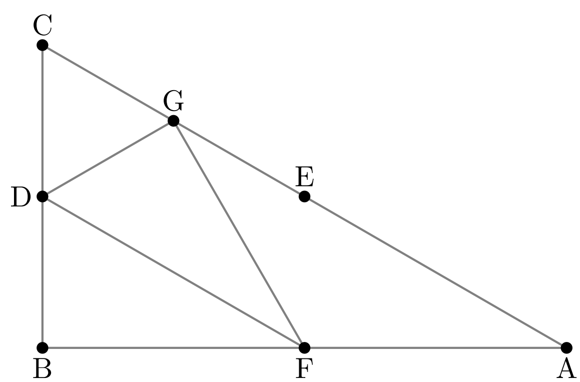


10. The figure below shows a semicircle inscribed in a right triangle.

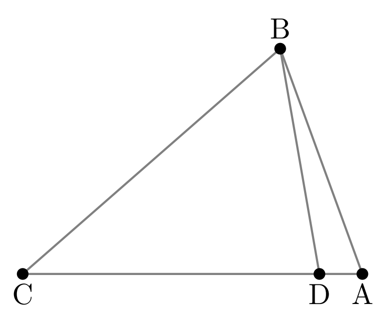


The triangle has legs of length 8 and 15. The semicircle is tangent to the two legs, and its diameter is on the hypotenuse. What is the radius of the semicircle? Express your answer as a fraction in simplest form.

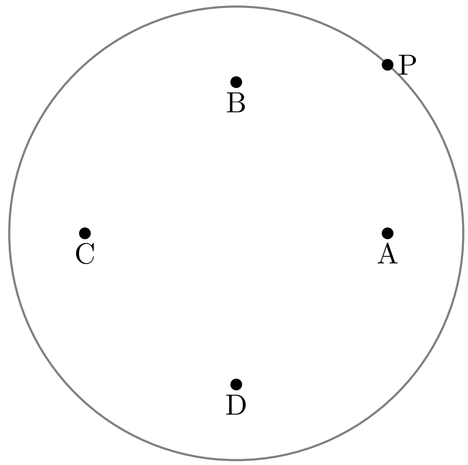
11. Let $\triangle ABC$ be a triangle with a right angle $\angle ABC$. Let D be the midpoint of $\overline{BC}$, let E be the midpoint of $\overline{AC}$, and let F be the midpoint of $\overline{AB}$. Let G be the midpoint of $\overline{EC}$. One of the angles of $\triangle DFG$ is a right angle. What is the least possible value of $\frac{\overline{BC}}{\overline{AG}}$? Express your answer as a fraction in simplest form.



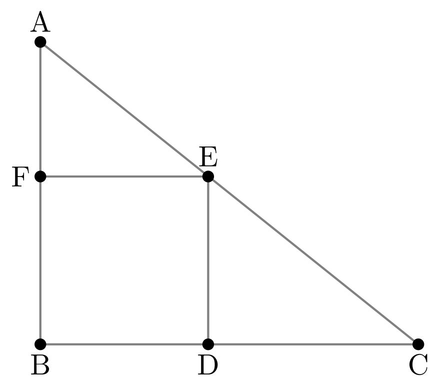
12. Let $\triangle ABC$ be a triangle with $\overline{AB} = 7$, $\overline{BC} = 8$, and $\overline{AC} = 9$. Point D is on side $\overline{AC}$ such that $\angle CBD$ has measure $45°$. What is the length of $\overline{BD}$? Express your answer in the form $m\sqrt{x} - n\sqrt{y}$, where m and n are positive integers and x and y are square-free positive integers.



13. Let $A = (2, 0)$, $B = (0, 2)$, $C = (-2, 0)$, and $D = (0, -2)$. Compute the greatest possible value of the product $\overline{PA} \cdot \overline{PB} \cdot \overline{PC} \cdot \overline{PD}$, where P is a point on the circle $x^2 + y^2 = 9$.

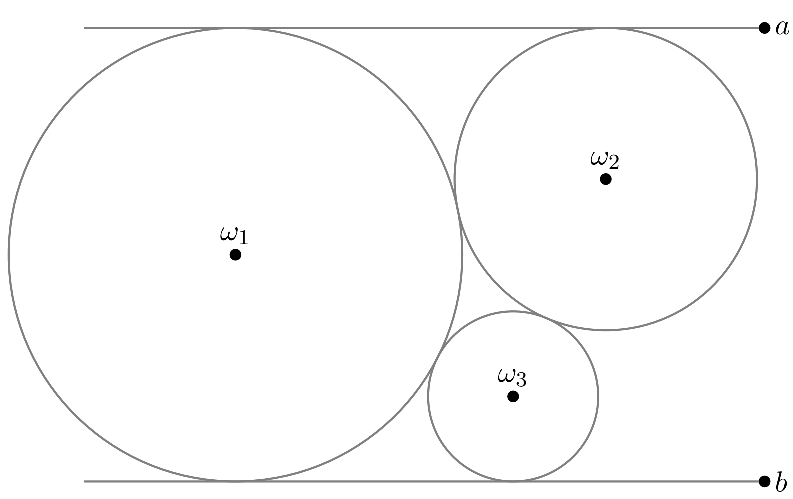


14. In the figure below, $BDEF$ is a square inscribed in $\triangle ABC$.



If $\frac{\overline{AB}}{\overline{BC}} = \frac{4}{5}$, what is the area of $BDEF$ divided by the area of $\triangle ABC$? Express your answer as a fraction in simplest form.

15. Circle ω_1 with radius 3 is inscribed in a strip S having border lines a and b. Circle ω_2 within S with radius 2 is tangent externally to both circles ω_1 and ω_2, and is also tangent to line b. Compute the radius of circle ω_3. Express your answer as a fraction in simplest form.



2 Sources

1. Math Prize for Girls 2009 Problem 5
2. Math Prize for Girls 2009 Problem 13
3. Math Prize for Girls 2010 Problem 6
4. Math Prize for Girls 2010 Problem 10
5. Math Prize for Girls 2010 Problem 11
6. Math Prize for Girls 2010 Problem 14
7. Math Prize for Girls 2011 Problem 5
8. Math Prize for Girls 2011 Problem 8
9. Math Prize for Girls 2011 Problem 9
10. Math Prize for Girls 2012 Problem 5
11. Math Prize for Girls 2012 Problem 10
12. Math Prize for Girls 2012 Problem 15
13. Math Prize for Girls 2015 Problem 11
14. Math Prize for Girls 2017 Problem 2
15. Math Prize for Girls 2017 Problem 17