

# Combinatorics Handout 2

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## 1 Problems

1. If you roll three fair, six-sided dice, what is the probability that the product of the results will be a multiple of 3?
2. Derek has 10 American coins in his pocket, summing to a total of 53 cents. If he randomly grabs 3 coins from his pocket, what is the probability that they're all different?
3. 15 people, including Luke and Matt, attend a Berkeley Math meeting. If Luke and Matt sit next to each other, a fight will break out. If they sit around a circular table, all positions equally likely, what is the probability that a fight doesn't break out?
4. How many ways can you arrange the letters of the word BERKELEY such that no two Es are next to each other?
5. Tom has 2012 blue cards, 2012 red cards, and 2012 boxes. He distributes the cards in such a way such that each box has at least 1 card. Sam chooses a box randomly, then chooses a card randomly. Suppose that Tom arranges the cards such that the probability of Sam choosing a blue card is maximized. What is this maximum probability?
6. A bag holds 6 coins. Three have tails on both sides, two have heads on both sides, and one has heads on one side and tails on the other. If you pick a coin at random and notice the only side you can see is heads, what is the probability that the other side is also a head?
7. How many solutions  $(x, y)$  in the positive integers are there to  $3x + 7y = 1337$ ?
8. Brandon is located at  $(3, 2)$ , and Chuck is located at  $(6, -5)$ . Brandon can only move unit distance to the right or down, and Chuck is stationary. In how many different ways can Brandon move to Chuck?
9. Four points  $A, B, C$ , and  $D$  are randomly chosen on a circle. What is the probability that  $AB$  and  $CD$  intersect inside the circle?
10. How many ways can you arrange the letters in MATHISHARD such that the permutation begins with MR?
11. There are 12 people labeled  $1, 2, \dots, 12$  working together on 12 missions, with persons  $1, 2, \dots, i$  working on the  $i$ th mission. There is exactly one spy among them. If the spy is not working on a mission, it will be a huge success, but if the spy is working on the mission, it will fail with probability  $\frac{1}{2}$ . Given that the first 11 missions succeed, and the 12th mission fails, what is the probability that person 12 is the spy?
12. In prokaryotes, translation of mRNA messages into proteins is most often initiated at start codons on the mRNA having the sequence AUG. Assume that the mRNA is single-stranded and consists of a sequence of bases, each described by a single letter, A, C, U, or G. Consider the set of

all pieces of bacterial mRNA six bases in length. How many such mRNA sequences have either no As or no Us?

**13.** Inside a LilacBall, you can find one of 7 different notes, each equally likely. Delcatty must collect all 7 notes in order to restore harmony and save Kanto from eternal darkness. What is the expected number of LilacBalls she must open in order to do so?

**14.** Link starts at the top left corner of an  $12 \times 12$  grid and wants to reach the bottom right corner. He can only move down or right. A turn is defined a down move immediately followed by a right move, or a right move immediately followed by a down move. Given that he makes exactly 6 turns, in how many ways can he reach his destination?

**15.** Katniss has an  $n$ -sided fair die which she rolls. If  $n > 2$ , she can either choose to let the value rolled be her score, or she can choose to roll a  $n - 1$  sided fair die, continuing the process. What is the expected value of her score assuming Katniss starts with a 6-sided die and plays to maximize this expected value.

## 2 Sources

1. Berkeley Math Tournament Individual Fall 2012 Problem 3
2. Berkeley Math Tournament Individual Fall 2012 Problem 10
3. Berkeley Math Tournament Individual Fall 2012 Problem 13
4. Berkeley Math Tournament Individual Fall 2012 Problem 15
5. Berkeley Math Tournament Individual Fall 2012 Problem 17
6. Berkeley Math Tournament Gambling Fall 2012 Problem 1
7. Berkeley Math Tournament Team Fall 2012 Problem 4
8. Berkeley Math Tournament Countdown Fall 2012 Problem 4
9. Berkeley Math Tournament Countdown Fall 2012 Problem 5
10. Berkeley Math Tournament Countdown Fall 2012 Problem 8
11. Berkeley Math Tournament Team Spring 2012 Problem 4
12. Berkeley Math Tournament Tournament Round 1 Spring 2012 Problem 3
13. Berkeley Math Tournament Tournament Round 2 Spring 2012 Problem 1
14. Berkeley Math Tournament Tournament Round 4 Spring 2012 Problem 6
15. Berkeley Math Tournament Championship Round Spring 2012 Problem 2