Combinatorics Handout 3

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1 Problems

1. You toss five coins one after another. What is the probability that you never get two consecutive heads or two consecutive tails?

2. Out of 100 customers at a market, 80 purchased oranges, 60 purchased apples, and 70 purchased bananas. What is the least possible number of customers who bought all three items?

3. Twelve distinct four-letter words can be formed by using each of the letters B, M, M, and T exactly once. What is the ninth word when these words are arranged in alphabetical order?

4. How many three digit even numbers are there with an even number of even digits?

5. Three boys, Bob, Charles, and Derek, and three girls, Alice, Elizabeth, and Felicia are all standing in one line. Bob and Derek are each adjacent to precisely one girl, while Felicia is next to two boys. If Alice stands before Charles, who stands before Elizabeth, determine the number of possible ways they can stand in a line.

6. A *spirited* integer is a positive number representable in the form $20^n + 13k$ for some positive integer n and any integer k. Determine how many *spirited* integers are less than 2013.

7. A finite set of distinct, nonnegative integers $\{a_1, ..., a_k\}$ is called *admissible* if the integer function $f(n) = (n+a_1)\cdots(n+a_k)$ has no common divisor over all terms; that is, $\gcd(f(1), f(2), ..., f(n)) = 1$ for any integer n. How many *admissible* sets only have members of value less than 10? $\{4\}$ and $\{0, 2, 6\}$ are such sets, but $\{4, 9\}$ and $\{1, 3, 5\}$ are not.

8. The pages of a book are consecutively numbered from 1 through 480. How many times does the digit 8 appear in this numbering?

9. A positive integer is said to be *binary-emulating* if its base three representation consists of only 0s and 1s. Determine the sum of the first 15 *binary-emulating* numbers.

10. Professor X can choose to assign homework problems from a set of problems labeled 1 to 30, inclusive. No two problems in his assignment can share a common divisor greater than 1. What is the maximum number of problems that Professor X can assign?

11. Alice and Bob play a game of Berkeley Ball, in which the first person to win four rounds is the winner. No round can end in a draw. How many distinct games can be played in which Alice is the winner? (Two games are said to be identical if either player wins/loses rounds in the same order in both games.)

12. How many ways are there to place 3 indistinguishable pegs on a 5×5 chessboard so that no two pegs lie in the same row or column?

13. Two boxes contain some number of red, yellow, and blue balls. The first box has 3 red, 4 yellow, and 5 blue balls, and the second box has 6 red, 2 yellow, and 7 blue balls. There are two

ways to select a ball from these boxes; one could first randomly choose a box and then randomly select a ball or one could put all the balls in the same box and simply randomly select a ball from there. How much greater is the probability of drawing a red ball using the second method than the first?

14. A coin is flipped until there is a head followed by two tails. What is the probability that this will take exactly 12 flips?

15. If I roll three fair 4-sided dice, what is the probability that the sum of the resulting numbers is relatively prime to the product of the resulting numbers?

2 Sources

1. Berkeley Math Tournament Individual Fall 2013 Problem 6
2. Berkeley Math Tournament Individual Fall 2013 Problem 8
3. Berkeley Math Tournament Individual Fall 2013 Problem 10
4. Berkeley Math Tournament Individual Fall 2013 Problem 12
5. Berkeley Math Tournament Individual Fall 2013 Problem 13
6. Berkeley Math Tournament Individual Fall 2013 Problem 18
7. Berkeley Math Tournament Individual Fall 2013 Problem 20
8. Berkeley Math Tournament Team Fall 2013 Problem 8
9. Berkeley Math Tournament Team Fall 2013 Problem 12
10. Berkeley Math Tournament Team Fall 2013 Problem 13
11. Berkeley Math Tournament Team Fall 2013 Problem 15
12. Berkeley Math Tournament Speed Fall 2013 Problem 59
13. Berkeley Math Tournament Individual Spring 2013 Problem 3
14. Berkeley Math Tournament Discrete Spring 2013 Problem 6
15. Berkeley Math Tournament Discrete Spring 2013 Problem 2