Combinatorics Handout #6

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1 Problems

1. On 5×5 grid of lattice points, every point is uniformly randomly colored blue, red, or green. Find the expected number of monochromatic triangles T with vertices chosen from the lattice grid, such that some two sides of T are parallel to the axis.

2. What is the number of nondecreasing positive integer sequences of length 7 whose last term is at most 9?

3. How many ways can we pick four 3-element subsets of $\{1, 2, ..., 6\}$ so that each pair of subsets share exactly one element?

4. How many five-card hands from a standard deck of 52 cards are full houses? A full house consists of 3 cards of one rank and 2 cards of another rank.

5. Three 3-legged (distinguishable) Stanfurdians take off their socks and trade them with each other. How many ways is this possible if everyone ends up with exactly 3 socks and nobody gets any of their own socks? All socks originating from the Stanfurdians are distinguishable from each other. All Stanfurdian feet are indistinguishable from other feet of the same Stanfurdian.

6. How many subsets (including the empty-set) of $\{1, 2..., 6\}$ do not have three consecutive integers?

7. Consider an urb containing 51 white and 50 black balls. Every turn, we randomly pick a ball, record the color of the ball, and then we put the ball back into the urn. We stop picking when we have recorded n black balls, where n is an integer randomly chosen from $\{1, 2, ..., 100\}$ What is the expected number of turns?

8. There are 2000 math students and 4000 CS students at Berkeley. If 5580 students are either math students or CS students, then how many of them are studying both math and CS?

9. I have 18 socks in my drawer, 6 colored red, 8 colored blue and 4 colored green. If I close my eyes and grab a bunch of socks, how many socks must I grab to guarantee there will be two pairs of matching socks?

10. Alice, Bob, and four other people sit themselves around a circular table. What is the probability that Alice does not sit to the left or right of Bob?

11. There are 20 indistinguishable balls to be placed into bins A, B, C, D, and E. Each bin must have at least 2 balls inside of it. How many ways can the balls be placed into the bins, if each ball must be placed in a bin?

12. A *gorgeous* sequence is a sequence of 1's and 0's such that there are no consecutive 1's. For instance, the set of all *gorgeous* sequences of length 3 is $\{[1, 0, 0], [1, 0, 1], [0, 1, 0], [0, 0, 1], [0, 0, 0]\}$. Determine the number of *gorgeous* sequences of length 7.

13. A coin is flipped 12 times. What is the probability that the total number of heads equals the total number of tails? Express your answer as a common fraction in lowest terms.

14. Debbie has six Pusheens: 2 pink ones, 2 gray ones, and 2 blue ones, where Pusheens of the same color are indistinguishable. She sells two Pusheens each to Alice, Bob, and Eve. How many ways are there for her to do so?

15. A *prim* number is a number that is prime if its last digit is removed. A *rime* number is a number that is prime if its first digit is removed. Determine how many numbers between 100 and 999 inclusive are both *prim* and *rime* numbers.

2 Sources

1. 2016 Berkeley Math Tournament Spring Individual Problem 9
2. 2016 Berkeley Math Tournament Spring Individual Problem 12
3. 2016 Berkeley Math Tournament Spring Individual Problem 15
4. 2016 Berkeley Math Tournament Spring Discrete Problem 3
5. 2016 Berkeley Math Tournament Spring Discrete Problem 4
6. 2016 Berkeley Math Tournament Spring Team Problem 9
7. 2016 Berkeley Math Tournament Spring Team Problem 13
8. 2017 Berkeley Math Tournament Fall Individual Problem 3
9. 2017 Berkeley Math Tournament Fall Individual Problem 8
10. 2017 Berkeley Math Tournament Fall Individual Problem 13
11. 2017 Berkeley Math Tournament Fall Individual Problem 18
12. 2017 Berkeley Math Tournament Fall Individual Problem 20
13. 2017 Berkeley Math Tournament Fall Team Problem 5
14. 2017 Berkeley Math Tournament Fall Team Problem 8
15. 2017 Berkeley Math Tournament Fall Team Problem 12