Combinatorics Handout # 7

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1 Problems

1. You have 9 colors of socks and 5 socks of each type of color. Pick two socks randomly. What is the probability that they are the same color?

2. Each BMT, every student chooses one of three focus rounds to take. Bob plans to attend BMT for the next 4 years and wants to figure out what focus round to take each year. Given that he wants to take each focus round at least once, how many ways can he choose which round to take each year?

3. How many subsets of $\{1, 2, ..., 9\}$ do not contain 2 adjacent numbers?

4. Let $S = \{1, 2, ...6\}$. How many functions $f : S \to S$ are there such that for all $s \in S$,

$$f^5(s) = f(f(f(f(f(s))))) = 1.$$

5. You enter an elevator on floor 0 of a building with some other people, and request to go to floor 10. In order to be efficient, it doesn't stop at adjacent floors (so, if it's at floor 0, its next stop cannot be floor 1). Given that the elevator will stop at floor 10, no matter what other floors it stops at, how many combinations of stops are there for the elevator?

6. Eric has a 9-sided die and Harrison has an 11-sided die. They each roll their respective die. Eric wins if he rolls a number greater than or equal to Harrison's number. What is the probability that Eric wins?

7. Ed writes the first 2018 positive integers down in order: $1, 2, 3, ..., 2018$. Then for each power of 2 that appears, he crosses out that number as well as the number 1 greater than that power of 2. After he is done, how many numbers are not crossed out?

8. The sequence $2, 3, 5, 6, 7, 8, 10, ...$ contains all positive integers that are not perfect squares. Find the 2018th term of the sequence.

9. A number is formed using the digits $\{2, 0, 1, 8\}$, using all 4 digits exactly once. Note that $0218 = 218$ is a valid number that can be formed. What is the probability that the resulting number is strictly greater than 2018?

10. A dice is labeled with the integers $1, 2, ..., n$ such that it is 2 times as likely to roll a 2 as it is a 1, 3 times as likely to roll a 3 as it is a 1, and so on. Suppose the probability of rolling an odd integer with the dice is $\frac{17}{35}$. Compute n.

11. Let S be the set of all 1000 element subsets of the set $\{1, 2, 3, ..., 2018\}$. What is the expected value of the minimum element of a set chosen uniformly at random from S?

12. A lattice point is a point (a, b) on the Cartesian plane where a and b are integers. Compute the number of lattice points in the interior and on the boundary of the triangle with vertices at $(0, 0), (0, 20)$, and $(18, 0)$.

13. Three distinct points are chosen uniformly at random from the vertices of a regular 2018-gon. What is the probability that the triangle formed by these points is a right triangle?

14. How many ways are there to partition 11 into a sum of an odd number of odd positive integers? Order does not matter, so $11 = 3 = 3 = 5$ and $11 = 3 + 5 + 3$ should be counted only once.

15. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

2 Sources

1. 2017 Berkeley Math Tournament Spring Discrete Problem 1
2. 2017 Berkeley Math Tournament Spring Discrete Problem 2
3. 2017 Berkeley Math Tournament Spring Discrete Problem 5
4. 2017 Berkeley Math Tournament Spring Discrete Problem 6
5. 2017 Berkeley Math Tournament Spring Team Problem 5
6. 2018 Stanford Math Tournament Spring General Problem 6
7. 2018 Stanford Math Tournament Spring General Problem 9
8. 2018 Stanford Math Tournament Spring General Problem 11
9. 2018 Stanford Math Tournament Spring Discrete Problem 1
10. 2018 Stanford Math Tournament Spring Discrete Problem 3
11. 2018 Stanford Math Tournament Spring Discrete Problem 7
12. 2018 Stanford Math Tournament Spring Team Problem 3
13. 2018 Stanford Math Tournament Spring Team Problem 6
14. 2018 Stanford Math Tournament Spring General Tiebreaker Problem 3
15. 2018 AMC 12B Problem 10