Combinatorics Handout #4

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1 Problems

1. Suppose we have 2013 piles of coins, with the i^{th} pile containing exactly i coins. We wish to remove the coins in a series of steps. In each step, we are allowed to take away coins from as many piles as we wish, but we have to take the same number of coins from each pile. We cannot take away more coins than a pile actually has. What is the minimum number of steps we have to take?

2. A coin is flipped until there is a head followed by two tails. What is the probability that this will take exactly 12 flips?

3. A number is called boxy if the number of its factors is a perfect square. Find the largest boxy number less than 200.

4. Alice, Bob, Clara, David, Eve, Fred, Greg, Harriet, and Isaac are on a committee. They need to split into three subcommittees of three people each. If not subcommittee can be all male or all female, how many ways are there to do this?

5. How many integers between 0 and 999 are not divisible by $7, 11$, or 13?

6. Compute the number of ways to make 50 cents using only pennies, nickels, dimes, and quarters.

7. Given that there are 168 primes with 3 digits or less, how many numbers between 1 and 1000 inclusive have a prime number of factors?

8. Your wardrobe contains two red socks, two green socks, two blue socks, and two yellow socks. It is currently dark right now, but you decide to pair up the socks randomly. What is the probability that none of the pairs are of the same color?

9. Fred and George are playing a game, in which Fred flips 2014 coins and George flips 2015 coins. Fred wins if he flips at least as many heads as George does, and George wins if he flips more heads than Fred does. Determine the probability that Fred wins.

10. Albert and Kevin are playing a game. Kevin has a 10% chance of winning any given round in the match. If Kevin wins the first game, he wins the match. If not, he requests that the match be extended to a best of 3. If he wins the best of 3, he wins the match. If not, then he requests the match be extended to a best of 5, and so forth. What is the probability that Kevin eventually wins the match? (A best of $2n + 1$ match consists of a series of rounds. The first person to reach $n + 1$ winning games wins the match)

11. Find the number of 5-digit n such that every digit of n is either $0, 1, 3$, or 4, and n is divisible by 15.

12. Find the number of functions from the set $\{1, 2, ..., 8\}$ to itself such that $f(f(x)) = x$ for all $1 \leq x \leq 8$.

13. Find the number of non-negative integer solutions (x, y, z) of the equation

$$xyz + xy + yz + zx + x + y + z = 2014.$$

14. How many proper subsets of $\{1, 2, 3, 4, 5, 6\}$ are there such that the sum of the elements in the subset is equal to twice a number in the subset?

15. Thomas, Olga, Ken, and Edward are playing the card game SAND. Each draws a card from a 52 card deck. What is the probability that each player gets a different rank and a different suit from the others?

2 Sources

1. Berkeley Math Tournament Discrete Spring 2013 Problem 3
2. Berkeley Math Tournament Discrete Spring 2013 Problem 6
3. Berkeley Math Tournament Individual Fall 2014 Problem 4
4. Berkeley Math Tournament Individual Fall 2014 Problem 11
5. Berkeley Math Tournament Team Fall 2014 Problem 3
6. Berkeley Math Tournament Team Fall 2014 Problem 4
7. Berkeley Math Tournament Team Fall 2014 Problem 12
8. Berkeley Math Tournament Team Fall 2014 Problem 19
9. Berkeley Math Tournament Individual Spring 2014 Problem 5
10. Berkeley Math Tournament Individual Spring 2014 Problem 15
11. Berkeley Math Tournament Discrete Spring 2014 Problem 2
12. Berkeley Math Tournament Individual Fall 2015 Problem 10
13. Berkeley Math Tournament Individual Fall 2015 Problem 17
14. Berkeley Math Tournament Team Fall 2015 Problem 6
15. Berkeley Math Tournament Team Fall 2015 Problem 8