Combinatorics Handout #5

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1 Problems

1. The boba shop sells four different types of milk tea, and William likes to get tea each weekday. If William refuses to have the same type of tea on successive days, how many different combinations could he get, Monday through Friday?

2. Suppose we list the decimal representations of the positive even numbers from left to right. Determine the 2015^{th} digit in the list.

3. Find the number of ways to partition a set of 10 elements, $S = \{1, 2, 3, ..., 10\}$ into two parts; that is, the number of unordered pairs $\{P, Q\}$ such that $P \cup Q = S$ and $P \cap Q = \emptyset$.

4. An integer is between 0 and 99999 (inclusive) is chosen, and the digits of its decimal representation are summed. What is the probability that the sum will be 19?

5. We have 10 boxes of different sizes, each one big enough to contain all the smaller boxes when put side by side. We may nest the boxes however we want (and how deeply we want), as long as we put smaller boxes in larger ones. At the end, all boxes should be directly or indirectly nested in the largest box. How many ways can we nest the boxes?

6. Alice is planning a trip from the Bay Area to one of 5 possible destinations (each of which is serviced by only 1 airport) and wants to book two flights, one to her destination and one returning. There are 3 airports within the Bay Area from which she may leave and to which she may return. In how many ways may she plan her flight itinerary?

7. Three balloon vendors each offer two types of balloons - one offers red & blue, one offers blue & yellow, and one offers yellow & red. I like each vendor the same, so I must buy 7 balloons from each. How many different triples (x, y, z) are there such that I could buy x blue, y yellow, and z red balloons?

8. There are 30 cities in the empire of Euleria. Every week, Martingale City runs a very well-known lottery. 900 visitors decide to take a trip around the empire, visiting a different city each week in some random order. 3 of these cities are inhabited by mathematicians, who will talk to all visitors about the laws of statistics. A visitor with this knowledge has probability 0 of buying a lottery ticket, else they have probability 0.5 of buying one. What is the expected number of visitors who will play the Martingale Lottery?

9. A fair 6-sided die is repeatedly rolled until a $1, 4, 5$, or 6 is rolled. What is the expected value of the product of all the rolls?

10. How many ways are there to place the numbers $2, 3, ..., 10$ in a 3×3 grid, such that any two numbers that share an edge are mutually prime?

11. Consider the set $S = \{1, 2, ..., 2015\}$. How many ways are there to choose 2015 distinct (possibly empty and possibly full) subsets $X_1, X_2, ..., X_{2015}$ of S such that X_i is strictly contained

in X_{i+1} for all $1 \leq i \leq 2014$?

12. You decide to flip a coin some number of times, and record each of the results. You stop flipping the coin once you have recorded either 20 heads, or 16 tails. What is the maximum number of times that you could have flipped the coin?

13. How many 8-digit positive integers have the property that the digits are strictly increasing from left to right? For instance, 12356789 is an example of such a number, while 12337889 is not.

14. How many different ways are there to arrange the letters $MILKTEA$ such that TEA is a contiguous substring?

For reference, the term "contiguous substring" means that the letters TEA appear in that order, all next to one another. For example, $MITEALK$ would be such a string, while $TMIELKA$ would not be.

15. Suppose you roll two fair 20-sided dice. What is the probability that their sum is divisible by 10?

2 Sources

1. 2015 Berkeley Math Tournament Spring Individual Problem 1
2. 2015 Berkeley Math Tournament Spring Individual Problem 2
3. 2015 Berkeley Math Tournament Spring Individual Problem 5
4. 2015 Berkeley Math Tournament Spring Individual Problem 8
5. 2015 Berkeley Math Tournament Spring Individual Problem 10
6. 2015 Berkeley Math Tournament Spring Discrete Problem 1
7. 2015 Berkeley Math Tournament Spring Discrete Problem 5
8. 2015 Berkeley Math Tournament Spring Discrete Problem 6
9. 2015 Berkeley Math Tournament Spring Team Problem 1
10. 2015 Berkeley Math Tournament Spring Team Problem 3
11. 2015 Berkeley Math Tournament Spring Team Problem 6
12. 2016 Berkeley Math Tournament Fall Individual Problem 2
13. 2016 Berkeley Math Tournament Fall Individual Problem 8
14. 2016 Berkeley Math Tournament Fall Individual Problem 10
15. 2016 Berkeley Math Tournament Fall Individual Problem 11