# Combinatorics Handout #8

Walker Kroubalkian

April 17, 2018

## 1 Problems

**1.** Consider a round table with six identical chairs. If four students and two teachers randomly take seats, what is the probability that two teachers will not sit next to each other?

**2.** There are three different French books and two different Spanish books. How many ways are there to arrange the books in a row on a shelf with all books of the same language grouped together?

**3.** Three points are randomly located on a circle. What is the probability that the shortest distance between each point is less than or equal to the radius of the circle?

**4.** How many two digit positive integers are multiples of 3 and/or 7?

**5.** How many ordered pairs $(x, y)$ of positive integers $x$ and $y$ satisfy the equation $3x + 5y = 80$?

**6.** Find the number of 10-tuples $(a_1, a_2, ..., a_{10})$ such that $a_i \in \{1, 2, 3\}$ for $1 \leq i \leq 10$, $a_i < a_{i+1}$ if $i = 1, 3, 5, 7, 9$ and $a_i > a_{i+1}$ if $i = 2, 4, 6, 8$.

**7.** Find the number of 5-tuples of positive integers $(x_1, x_2, x_3, x_4, x_5)$ such that $x_1 = x_5, x_i \neq x_{i+1}$ for $i = 1, 2, 3, 4,$, and $x_i + x_{i+1} \leq 6$ for $i = 1, 2, 3, 4$.

**8.** Find the number of functions $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ such that for $k = 1, 2, 3, 4$, $f(k + 1) \leq f(k) + 1$.

**9.** Let $q(n)$ be the number of ways to express $n$ as a sum of two positive integers, using each of them at least once. For example, since $5 = 4 + 1 = 3 + 2 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 1 + 1 + 1$, we have $q(5) = 5$. Find the number of positive integers $n$ such that $n \leq 100$, $n \equiv 3 \pmod 4$, and $q(n) \equiv 0 \pmod 2$.

**10.** In a $2 \times 6$ matrix, we want to fill 1 or 2 in each term. Also, for $i = 1, 2, 3, 4, 5, 6$, define $c_i$ as the product of the terms in the $i$th column. How many ways are there to fill the terms so that

$$\sum_{i=1}^{6} c_i \equiv 0 \pmod 2?$$

**11.** In the set $\{1, 2, 3, ..., 8\}$, how many subsets contain 4 consecutive integers?

**12.** In a regular 20-gon with 1 as the length of all sides, pick 5 points to make a pentagon. How many pentagons have all of its sides larger than 2? If two pentagons are the same when rotated, they are still considered to be different.

**13.** We want to choose 8 people out of 20 people who are sitting in a circle. We do not want to choose two people who are next to each other. Calculate how many ways are possible.

**14.** How many 5-digit numbers are there such that all digits are either $1, 2, 3$, or 4 and no two digits next to each other differ by 1?

**15.** Find the number of subsets of $\{1, 2, ..., 23\}$ such that the number of elements is 11 and the sum of the elements is 194.

## 2 Sources

**1.** KSEA National Mathematics Competition 2007 11th Grade Problem 2 (Korea)
**2.** KSEA National Mathematics Competition 2007 10th Grade Problem 11 (Korea)
**3.** KSEA National Mathematics Competition 2007 10th Grade Problem 15 (Korea)
**4.** KSEA National Mathematics Competition 2007 9th Grade Problem 7 (Korea)
**5.** KSEA National Mathematics Competition 2007 9th Grade Problem 10 (Korea)
**6.** Korean Mathematical Olympiad First Round 2015 Problem 2
**7.** Korean Mathematical Olympiad First Round 2015 Problem 7
**8.** Korean Mathematical Olympiad First Round 2015 Problem 10
**9.** Korean Mathematical Olympiad First Round 2015 Problem 12
**10.** Korean Mathematical Olympiad First Round 2014 Problem 2
**11.** Korean Mathematical Olympiad First Round 2014 Problem 5
**12.** Korean Mathematical Olympiad First Round 2014 Problem 10
**13.** Korean Mathematical Olympiad First Round 2013 Problem 1
**14.** Korean Mathematical Olympiad First Round 2013 Problem 12
**15.** Korean Mathematical Olympiad First Round 2012 Problem 1 (Adapted)