Algebra Handout 2

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1 Problems

1. You have a bag containing 3 types of pens: red, green, and blue. 30% of the pens are red pens, and 20% are green pens. If, after you add 10 blue pens, 60% of the pens are blue pens, how many green pens did you start with?

2. Bacon, Meat, and Tomato are dealing with paperwork. Bacon can fill out 5 forms in 3 minutes, Meat can fill out 7 forms in 5 minutes, and Tomato can staple 3 forms in 1 minute. A form must be filled out and stapled together (in either order) to complete it. How long will it take them to complete 105 forms?

3. What is the volume of a cylinder whose radius is equal to its height and whose surface area is numerically equal to its volume?

4. Given for some real a, b, c, d,

$$P(x) = ax^4 + bx^3 + cx^2 + dx$$

$$P(-5) = P(-2) = P(2) = P(5) = 1$$

Find P(10).

5. Allison wants to bake a pie, so she goes to the discount market with a handful of dollar bills. The discount market sells different fruit each for some integer number of cents and does not add tax to purchases. She buys 22 apples and 7 boxes of blueberries, using all of her dollar bills. When she arrives back home, she realizes she needs more fruit, though, so she grabs her checkbook and heads back to the market. This time, she buys 31 apples and 4 boxes of blueberries, for a total of 60 cents more than her last visit. Given she spent less than 100 dollars over the two trips, how much (in dollars) did she spend on her first trip to the market?

6. Ed, Fred, and George are playing on a see-saw that is slightly off center. When Ed sits on the left side and George, who weighs 100 pounds, sits on the right side, they are perfectly balanced. Similarly, if Fred, who weighs 400 pounds, sits on the left and Ed sits on the right, they are also perfectly balanced. Assuming the see-saw has negligible weight, what is the weight of Ed, in pounds?

7. Let $f(x) = 6x + 7$ and $g(x) = 7x + 6$. Find the value of a such that $g^{-1}(f^{-1}(g(f(a)))) = 1$.

8. Let $\{a_n\}_{n=1}^{\infty}$ be an arithmetic progression with $a_1 > 0$ and $5a_{13} = 6a_{19}$. What is the smallest integer n such that $a_n < 0$?

9. Simplify

$$[\log_{xyz}(x^z)][1 + \log_x y + \log_x z]$$

where $x = 628, y = 233, z = 340$.

10. The roots of the equation $x^3 + ax^2 + bx + c = 0$ are three consecutive integers. Find the maximum value of $\frac{a^2}{b+1}$.

11. Denote $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$. What is $144169 * S_{144169} - (S_1 + S_2 + \cdots + S_{144168})$?

12. Given $f(x) = \frac{3}{x-1}$, then express $\frac{9(x^2-2x+1)}{x^2-8x+16}$ entirely in terms of $f(x)$. In other words, x should not be in your answer, only $f(x)$.

13. Suppose that $f(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, and that $f(1) = f(2) = f(3) = f(4) = f(5) = f(6) = 7$. What is a?

14. There are two non-consecutive positive integers a, b such that $a^2 - b^2 = 291$. Find a and b.

15. Let r_1, r_2, r_3 be the roots of $f(x) = x^3 - 6x^2 - 5x + 22$. Find $\frac{1}{r_1 r_2} + \frac{1}{r_2 r_3} + \frac{1}{r_3 r_1}$.

2 Sources

1. 2012 Berkeley Math Tournament Fall Individual Problem 5
2. 2012 Berkeley Math Tournament Fall Individual Problem 7
3. 2012 Berkeley Math Tournament Fall Individual Problem 12
4. 2012 Berkeley Math Tournament Fall Individual Problem 20
5. 2012 Berkeley Math Tournament Fall Individual Problem 18
6. 2012 Berkeley Math Tournament Fall Team Problem 1
7. 2012 Berkeley Math Tournament Fall Team Problem 6
8. 2012 Berkeley Math Tournament Spring Individual Problem 1
9. 2012 Berkeley Math Tournament Spring Tournament Round 1 Problem 2
10. 2012 Berkeley Math Tournament Spring Tournament Round 3 Problem 4
11. 2012 Berkeley Math Tournament Spring Tournament Round 4 Problem 1
12. 2012 Berkeley Math Tournament Spring Tournament Round 5 Problem 3
13. 2012 Berkeley Math Tournament Spring Tournament Championship Round Problem 3
14. 2013 Berkeley Math Tournament Fall Team Problem 10
15. 2013 Berkeley Math Tournament Fall Speed Problem 89