Algebra Handout 1

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1 Problems

1. Compute the unique positive integer that, when squared, is equal to six more than five times itself.

2. An infinite geometric sequence has a first term of 12, and all terms in the sequence sum to 9. Compute the common ratio between consecutive terms of the geometric sequence.

3. Alice and Bob are painting a house. If Alice and Bob do not take any breaks, they will finish painting the house in 20 hours. If however, Bob stops painting once the house is half-finished, then the house takes 30 hours to finish. Given that Alice and Bob paint at a constant rate, compute how many hours it will take for Bob to paint the entire house if he does it by himself.

4. Compute $9^6 + 6 \cdot 9^5 + 15 \cdot 9^4 + 20 \cdot 9^3 + 15 \cdot 9^2 + 6 * 9$.

5. Let x_1 and x_2 be the roots of $x^2 - x - 2014$, with $x_1 < x_2$. Let x_3 and x_4 be the roots of $x^2 - 2x - 2014$, with $x_3 < x_4$. Compute $(x_4 - x_2) + (x_3 - x_1)$.

6. Robin goes birdwatching one day. He sees three types of birds: penguins, pigeons, and robins. $\frac{2}{3}$ of the birds he sees are robins. $\frac{1}{8}$ of the birds he sees are penguins. He sees exactly 5 pigeons. How many robins does Robin see?

7. A tree has 10 pounds of apples at dawn. Every afternoon, a bird comes and eats x pounds of apples. Overnight, the amount of food on the tree increases by 10%. What is the maximum value of x such that the bird can sustain itself indefinitely on the tree without the tree running out of food?

8. What is the greatest possible value of c such that $x^2 + 5x + c = 0$ has at least one real solution?

9. Caroline wants to plant 10 trees in her orchard. Planting n apple trees requires n^2 square meters, planting n apricot trees requires $5n$ square meters, and planting n plum trees requires n^3 square meters. If she is committed to growing only apple, apricot, and plum trees, what is the least amount of space, in square meters, that her garden will take up?

10. Let a and b be the solutions to $x^2 - 7x + 17 = 0$. Compute $a^4 + b^4$.

11. Nick is a runner, and his goal is to complete four laps around a circuit at an average speed of 10 mph. If he completes the first three laps at a constant speed of only 9 mph, what speed does he need to maintain in miles per hour on the fourth lap to achieve his goal?

12. Given that $f(x) + 2f(8 - x) = x^2$ for all real x, compute $f(2)$.

13. Karl likes the number 17. His favorite polynomials are monic quadratics with integer coefficients such that 17 is a root of the quadratic and the roots differ by no more than 17. Compute the sum of the coefficients of all of Karls favorite polynomials. (A monic quadratic is a quadratic polynomial whose x^2 term has a coefficient of 1.)

14. For exactly two real values of b, b_1 and b_2, the line $y = bx - 17$ intersects the parabola $y = x^2 + 2x + 3$ at exactly one point. Compute $b_1^2 + b_2^2$.

15. Compute the minimum possible value of $(x-1)^2 + (x-2)^2 + (x-3)^2 + (x-4)^2 + (x-5)^2$ for real values of x.

2 Sources

1. 2014 Stanford Math Tournament General Problem 1
2. 2014 Stanford Math Tournament General Problem 12
3. 2014 Stanford Math Tournament Algebra Problem 1
4. 2014 Stanford Math Tournament Algebra Problem 2
5. 2014 Stanford Math Tournament Algebra Problem 3
6. 2013 Stanford Math Tournament General Problem 1
7. 2013 Stanford Math Tournament General Problem 9
8. 2013 Stanford Math Tournament General Problem 12
9. 2013 Stanford Math Tournament General Problem 18
10. 2013 Stanford Math Tournament General Problem 23
11. 2013 Stanford Math Tournament Algebra Problem 1
12. 2013 Stanford Math Tournament Algebra Problem 4
13. 2013 Stanford Math Tournament Algebra Problem 3
14. 2013 Stanford Math Tournament Algebra Problem 5
15. 2012 Stanford Math Tournament Algebra Problem 1