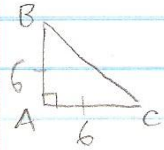
1. 

$A = 18\text{cm}^2$

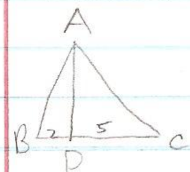
2. $11+15>k \cap 11+k>15 \Rightarrow 4<k<26$

121
225
346

$k^2+11^2<15^2 \Rightarrow k<11$

$11^2+15^2<k^2 \Rightarrow k>18$

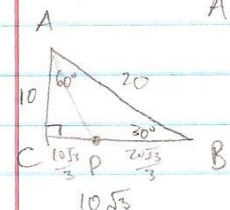
$4<k<11 \cup 18<k<26 = 13$ integers

3. 

$\frac{AB}{AC} = \frac{2}{5}$ $AB+AC=10$ let $AB=x$

$\frac{2}{5} = \frac{x}{10-x}$ $x = \frac{20}{7}$

$AB = \frac{20}{7}$; $AC = \frac{50}{7}$

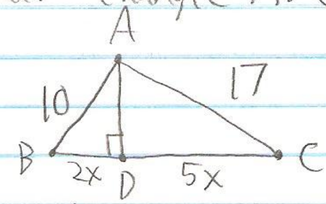
4. 

$\Rightarrow$ 30-60-90

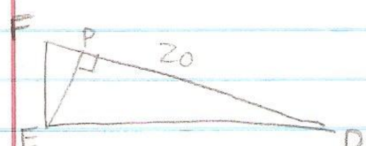
Since $\frac{10}{20} = \frac{10\sqrt{3}/3}{20\sqrt{3}/3}$, AP bisects $\angle A$

$30° - 30° = \boxed{0}$

5. We have our triangle ABC

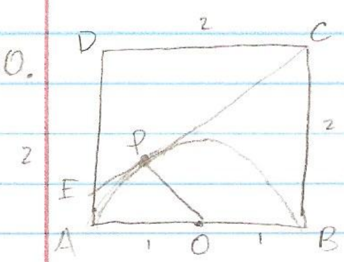


Looking for Pythagorean triples to go with our known values we come up with 8-15-17 and 6-8-10 for triangles ACD and ABD, respectively. This gives $BD=6$, $DC=15$, which satisfies $BD:DC=2:5$, so $\boxed{AD=8}$ QED

8. 

Draw altitude EP from E.

$EP = \frac{DF}{4} = 5$. $\frac{1}{2}bh = \frac{1}{2}\cdot 20 \cdot 5 = \boxed{50}$

10. 

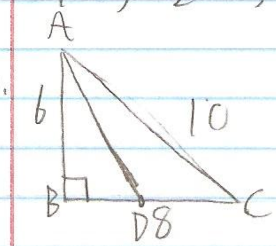
Draw $\overline{OP}$ and ~~notice~~ $\overline{OC}$ and notice $\triangle CPO$ and $\triangle CBO$ are congruent, So $CP=2$. Also draw $\overline{EO}$ and notice $\triangle EAO$ and $\triangle EPO$ are congruent. Let $EP=EA=x$. Using Pythagorean theorem on $\triangle CDE$, $2^2+(2-x)^2=(2+x)^2$. $x=\frac{1}{2}$, so $PE=\frac{1}{2} \Rightarrow CE=\boxed{\frac{5}{2}}$

11. Let s_1 be the side length of the smaller triangle and s_2 the side length of the larger. By the equilateral triangle area formula, $16\left(\frac{s_1^2\sqrt{3}}{4}\right)=\frac{s_2^2\sqrt{3}}{4}$, so

$$16s_1^2=s_2^2 \qquad (1)$$

$$4s_1=s_2 \qquad (2)$$

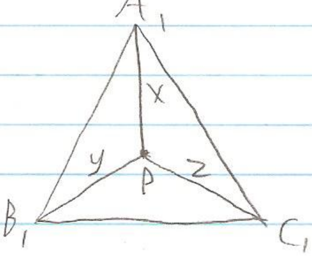
We also know that $3s_1+3s_2=45$. Substituting gives $s_1=3$, $s_2=12$, so the desired quantity is $\frac{(12)^2\sqrt{3}}{4}=\boxed{36\sqrt{3}}$

16. 

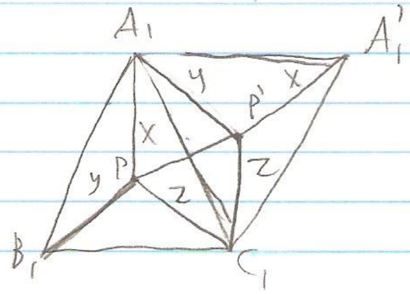
By Angle Bisector Theorem, $\frac{BD}{6}=\frac{DC}{10}$, and from the diagram, $BD+DC=8$. Solving for BD gives $BD=3$, so by the Pythagorean Theorem, $AD^2=6^2+3^2=\boxed{45}$

17. By the triangle inequality, $AB+BC>AC$, ~~the~~ We start by testing the smallest value of AB giving 1, 191, 192. This does not satisfy $AB+BC>AC$, ~~the~~ 2 also cannot satisfy the inequality with distinct integers, so 3, 190, 191 is the largest possible difference $\Rightarrow$ 188.

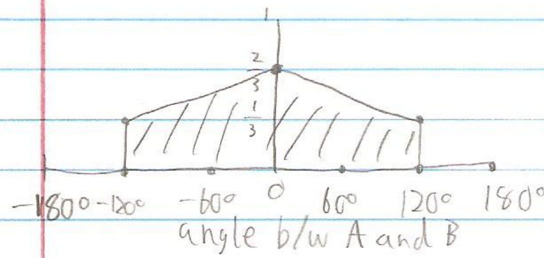
19. Consider equilateral triangle $A_1B_1C_1$ with arbitrary point P in its interior:

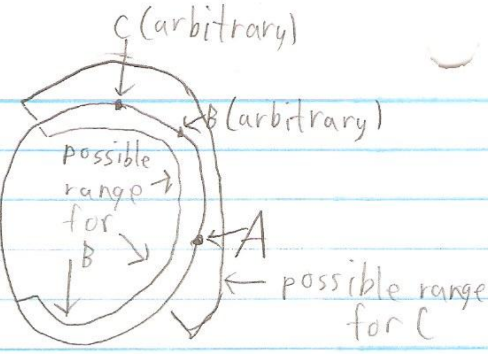


We know $x = BC$, $y = AC$, and $z = AB$. If we rotate $A_1B_1C_1$ $60°$ clockwise about C_1 to $A_1'A_1C_1$, we have:



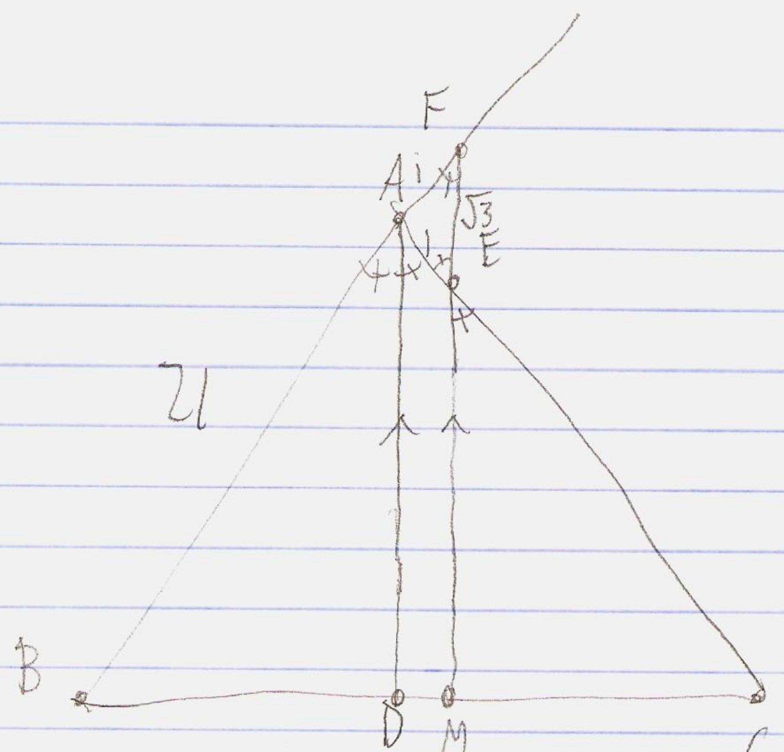
If we connect P to its image P', observe that triangle PC_1P' is equilateral (because $\angle PC_1P' = 60°$ and $PC_1 = P'C_1 = z$), so $PP' = z$ and $\Delta A_1PP' \cong \Delta ABC$ by SSS congruence. Observe that $\angle B_1A_1A_1' = 120°$, so for any set of points A, B, C on the circle, we must $\angle ACB < 120°$. Also, by symmetry, we must also have $\angle ABC < 120°$, $\angle BAC < 120°$. We now need to determine the probability that in ΔABC, all angles are less than $120°$, which we can accomplish through complementary counting. Arbitrarily choose point A on the circle. For triangle ABC to not fulfill the condition, A, B, and C must lie on the same $120°$ arc, so the number of possible positions for C is restricted by our choice of B. Since the possible positions are continuous, not discreet, we use geometric probability:





The area of the shaded region is our desired value, which can easily be shown to be $\frac{1}{3}$. Thus, the answer to the problem is $1-\frac{1}{3}=\frac{2}{3}$, so $2+3=\boxed{5}$

20.



Because FM is parallel to l, $\angle BAD$ is congruent to $\angle BFM$. By definition, $\angle BAD$ is congruent to $\angle DAC$. Because AD is parallel to FM, $\angle BAD$ is congruent to $\angle MEC$ which is congruent to $\angle AEF$. This means that ΔAFE is isosceles. Using 30-60-90 triangles, we find that $\angle FAE = 120°$. This tells us that $\angle BAC = 60°$. Let $AC = x$ and $BC = y$. We have that $MC = \frac{y}{2}$, and $DC = \frac{xy}{x+21}$. By similar triangles we have that

$$\frac{x-1}{x} = \frac{\left(\frac{y}{2}\right)}{\left(\frac{xy}{x+21}\right)} \longrightarrow \frac{x-1}{x} = \frac{x+21}{2x}.$$ Solving tells us

that $x = 23$. Using the Law of Cosines tells us that

$$BC^2 = 21^2 + 23^2 - 2 \cdot 21 \cdot 23 \cos 60 \rightarrow BC = \sqrt{487},$$

When $AC < AB$, we can use a similar method to get $B = \sqrt{403}$. $487 + 403 = 890$